Name:

AP Calculus BC

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Epsilon-Delta Limit Proofs Practice

Evaluate the limit and then write an $\varepsilon - \delta$ proof for the limit statement. If you need to, do your work on another paper and write the proof here.

1. $\lim\limits_{x\to 2}(3x+2) = 8$

Work: $|(3x+2)-8|<\varepsilon \Rightarrow |3x-6|<\varepsilon \Rightarrow |3(x-2)|<\varepsilon$

$|3||x-2|<\varepsilon \Rightarrow 3|x-2|<\varepsilon \Rightarrow |x-2|<\frac{\varepsilon}{3}$

Proof: let $\varepsilon>0$ be given. Let $\delta=\frac{\varepsilon}{3}$

So if $0<|x-2|<\delta$ then $|x-2|<\frac{\varepsilon}{3} \Leftrightarrow 3|x-2|<\varepsilon$

$\Leftrightarrow |3||x-2|<\varepsilon \Leftrightarrow |3(x-2)|<\varepsilon \Leftrightarrow |3x-6|<\varepsilon$

$\Leftrightarrow |(3x+2)-8|<\varepsilon$ $\therefore$ $\lim\limits_{x\to 2}(3x+2)=8$

2. $\lim\limits_{x\to 4}\left(4-\frac{x}{2}\right) = 2$

Work: $|(4-\frac{x}{2})-2|<\varepsilon \Rightarrow |2-\frac{x}{2}|<\varepsilon \Rightarrow |\frac{-1}{2}(-4+x)|<\varepsilon$

$\Rightarrow |\frac{-1}{2}||x-4|<\varepsilon \Rightarrow \frac{1}{2}|x-4|<\varepsilon \Rightarrow |x-4|<2\varepsilon$

Proof: let $\varepsilon>0$ be given. Let $\delta=2\varepsilon$

So if $0<|x-4|<\delta$ then $|x-4|<2\varepsilon \Leftrightarrow \frac{1}{2}|x-4|<\varepsilon \Leftrightarrow$

$|\frac{-1}{2}||x-4|<\varepsilon \Leftrightarrow |\frac{-1}{2}(x-4)|<\varepsilon \Leftrightarrow |2-\frac{1}{2}x|<\varepsilon \Leftrightarrow$

$|(4-\frac{1}{2}x)-2|<\varepsilon$ $\therefore$ $\lim\limits_{x\to 4}(4-\frac{x}{2})=2$

3. $\lim\limits_{x\to 4}(x+2) = 6$

Work: $|(x+2)-6|<\varepsilon \Rightarrow |x-4|<\varepsilon$

Proof: let $\varepsilon>0$ be given. Let $\delta=\varepsilon$

So if $0<|x-4|<\delta$ then $|x-4|<\varepsilon \Leftrightarrow |(x+2)-6|<\varepsilon$

$\therefore$ $\lim\limits_{x\to 4}(x+2)=6$

4. $\lim\limits_{x\to -3}(2x+5) = -1$

Work: $|(2x+5)-(-1)|<\varepsilon \Rightarrow |2x+6|<\varepsilon \Rightarrow |2(x+3)|<\varepsilon$

$\Rightarrow |2||x+3|<\varepsilon \Rightarrow 2|x+3|<\varepsilon \Rightarrow |x+3|<\frac{\varepsilon}{2}$

$\Rightarrow |x-(-3)|<\frac{\varepsilon}{2}$

Proof: let $\varepsilon>0$ be given. Let $\delta=\frac{\varepsilon}{2}$

So if $0<|x-(-3)|<\delta$ then $|x-(-3)|<\frac{\varepsilon}{2}$

$\Leftrightarrow |x+3|<\frac{\varepsilon}{2} \Leftrightarrow 2|x+3|<\varepsilon \Leftrightarrow |2||x+3|<\varepsilon$

$\Leftrightarrow |2(x+3)|<\varepsilon \Leftrightarrow |2x+6|<\varepsilon \Leftrightarrow |(2x+5)+1|<\varepsilon$

$\Leftrightarrow |(2x+5)-(-1)|<\varepsilon$ $\therefore$ $\lim\limits_{x\to -3}(2x+5)=-1$

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5. $\lim_{x \to -4}(\frac{1}{2}x-1) = -3$

Work: $|(\frac{1}{2}x-1)-(-3)|<\varepsilon \Rightarrow |(\frac{1}{2}x-1)+3|<\varepsilon$
$\Rightarrow |\frac{1}{2}x+2|<\varepsilon \Rightarrow |\frac{1}{2}(x+4)|<\varepsilon \Rightarrow |\frac{1}{2}||x+4|<\varepsilon$
$\Rightarrow \frac{1}{2}|x+4|<\varepsilon \Rightarrow |x+4|<2\varepsilon \Rightarrow |x-(-4)|<2\varepsilon$

Proof: Let $\varepsilon>0$ be given. Let $\delta=2\varepsilon$
So if $0<|x-(-4)|<\delta$ then $|x-(-4)|<2\varepsilon$
$\Leftrightarrow |x+4|<2\varepsilon \Leftrightarrow \frac{1}{2}|x+4|<\varepsilon \Leftrightarrow |\frac{1}{2}||x+4|<\varepsilon$
$\Leftrightarrow |\frac{1}{2}(x+4)|<\varepsilon \Leftrightarrow |\frac{1}{2}x+2|<\varepsilon \Leftrightarrow |(\frac{1}{2}x-1)+3|<\varepsilon$
$\Leftrightarrow |(\frac{1}{2}x-1)-(-3)|<\varepsilon \quad \therefore \lim_{x\to -4}(\frac{1}{2}x-1)=-3$

6. $\lim_{x \to 1}(\frac{2}{5}x+7) = \frac{37}{5}$

Work: $|(\frac{2}{5}x+7)-\frac{37}{5}|<\varepsilon \Rightarrow |(\frac{2}{5}x+\frac{35}{5})-\frac{37}{5}|<\varepsilon$
$\Rightarrow |\frac{2}{5}x-\frac{2}{5}|<\varepsilon \Rightarrow |\frac{2}{5}(x-1)|<\varepsilon \Rightarrow |\frac{2}{5}||x-1|<\varepsilon$
$\Rightarrow \frac{2}{5}|x-1|<\varepsilon \Rightarrow |x-1|<\frac{5}{2}\varepsilon$

Proof: Let $\varepsilon>0$ be given. Let $\delta=\frac{5}{2}\varepsilon$
So if $0<|x-1|<\delta$ then $|x-1|<\frac{5}{2}\varepsilon \Leftrightarrow \frac{2}{5}|x-1|<\varepsilon$
$\Leftrightarrow |\frac{2}{5}||x-1|<\varepsilon \Leftrightarrow |\frac{2}{5}(x-1)|<\varepsilon \Leftrightarrow |\frac{2}{5}x-\frac{2}{5}|<\varepsilon$
$\Leftrightarrow |(\frac{2}{5}x+\frac{35}{5})-\frac{37}{5}|<\varepsilon \Leftrightarrow |(\frac{2}{5}x+7)-\frac{37}{5}|<\varepsilon$
$\therefore \lim_{x\to 1}(\frac{2}{5}x+7)=\frac{37}{5}$

Prove the statement using the ϵ, δ definition of limit: $\lim_{x \to 2} \frac{x^2+x-6}{x-2} = 5$

Work: $\left|\frac{x^2+x-6}{x-2}-5\right|<\varepsilon \Rightarrow \left|\frac{(x+3)(x-2)}{(x-2)}-5\right|<\varepsilon \Rightarrow |(x+3)-5|<\varepsilon$

ok b/c don't care about $x=2$

$\Rightarrow |x-2|<\varepsilon$

Proof: Let $\varepsilon>0$ be given. Let $\delta=\varepsilon$
So if $0<|x-2|<\delta$ then $|x-2|<\varepsilon \Leftrightarrow |(x+3)-5|<\varepsilon \Leftrightarrow \left|\frac{(x+3)(x-2)}{(x-2)}-5\right|<\varepsilon$
$\Leftrightarrow \left|\frac{x^2+x-6}{x-2}-5\right|<\varepsilon \quad \therefore \lim_{x\to 2}\frac{x^2+x-6}{x-2}=5$