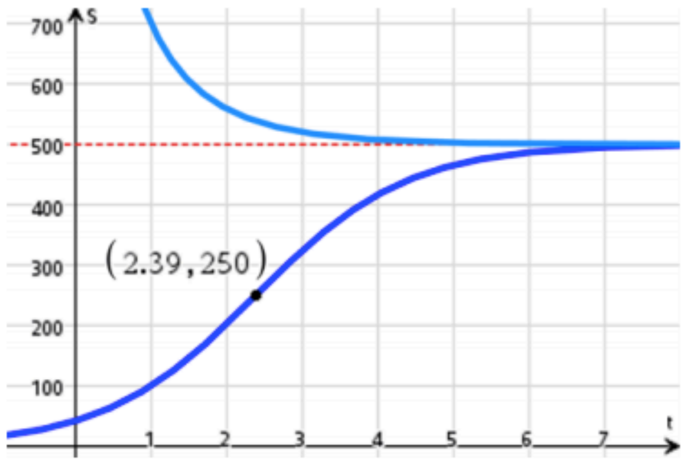
BC Extra Practice 2

Logistic Growth



What You Need to Know	
Logistic differential equation	$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$ or $\frac{dy}{dt} = ky(L - y)$
End behavior and carrying capacity	$\lim_{t \to \infty} y = L$
Fastest growth rate	when $y = \frac{L}{2}$
Concave up: y increasing at an increasing rate	$\frac{d^2y}{dt^2} > 0$ when $0 < y < \frac{L}{2}$
Concave down: y increasing at a decreasing rate	$\frac{d^2y}{dt^2} < 0$ when $\frac{L}{2} > y < L$
Increasing behavior	$\frac{dy}{dt} > 0$ for all t

3. The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation dP/dt = 0.0002P(1200 – P), where t is measured in years.

a. What is the carrying capacity for bears in this wildlife preserve?

b. What is the bear population when the population is growing the fastest?

c. What is the rate of change of the population when it is growing the fastest?

Given the differential equation $\frac{dy}{dx}=\frac{1}{x+2}$ and $y(0)=1$. Find an approximation of $y(1)$ using Euler's Method with two steps and step size $\Delta x=0.5$.

Given the differential equation $\frac{dy}{dx}=x+y$ and $y(1)=3$. Find an approximation of $y(2)$ using Euler's Method with two equal steps.

Assume that *f* and *f '* have the values given in the table. Use Euler's Method to approximate the value of *f*(4.4).

x	4	4.2	4.4
f'(*x*)	-0.5	-0.3	-0.1
f(*x*)	2		

x	0	1	4	6	8
$f'(x)$	$\sqrt{8}$	$\sqrt{3}$	0	$\sqrt{3}$	2

BC 5: The function f is twice differentiable for all real values with $f''(0) = -\frac{3}{8\sqrt{2}}$. Selected values of f', the derivative of f, are given in the table above. The arc length of the function $f(x)$ from 0 to x can be represented by the function S, defined by $S(x) = \int_0^x \sqrt{1 + [f'(t)]^2}\,dt$.

(a) Using a left Riemann sum with the four subintervals indicated in the table, approximate the arc length of the function $f(x)$ from $x = 0$ to $x = 8$.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size to approximate $S(8)$. Show the work that leads to your answer.

(c) Let $P_2(x)$ be the second degree Maclaurin polynomial for $S(x)$. Find $P_2(x)$ and use it to approximate $S(8)$.