Extra BC Practice

x	-1	0	1	2	3
f(x)	2	4	-3	2	0
f'(x)	3	-3	2	1	2
f''(x)	5	2	0	3	1
$f^3(x)$	-4	-1	7	9	1

a. Find $\int_0^3 2xf^3(x)\,dx$

$u = 2x \quad dv = f^3(x)dx$ Integration by Parts

$du = 2dx \quad v = f''(x)$

$\int_0^3 2xf^3(x)dx = 2xf''(x)\Big|_0^3 - \int_0^3 2f''(x)dx = [6\cdot f''(3) - 0] - 2\int_0^3 f''(x)dx$

$= [6\cdot 1] - 2[f'(x)]\Big|_0^3 = 6 - 2[f'(3) - f'(0)]$

$= 6 - 2[2+3]$

$= -4$

b. Write the third degree Taylor polynomial for f about x=2 and use it to approximate f(2.1).

$f(2) = 2$

$f'(2) = 1$

$f''(2) = 3$

$f^3(2) = 9$

$T_3(x) = 2 + 1(x-2) + \frac{3(x-2)^2}{2!} + \frac{9(x-2)^3}{3!}$

$f(2.1) \approx T_3(2.1) = 2 + (0.1) + \frac{3(0.1)^2}{2!} + \frac{9(0.1)^3}{3!}$

c. If $g(x) = \int_0^{\sin(x)} f(t)\,dt$, find the value of g'(0). Then find the value of g''(0).

$g'(x) = f(\sin(x))\cdot\cos(x)$

$g'(0) = f(\sin(0))\cdot\cos(0)$

$g'(0) = f(0)\cdot 1 = \boxed{4}$

chain rule

$g''(x) = \underbrace{f(\sin x)\cdot(-\sin x) + f'(\sin x)\cdot\cos x}_{\text{product rule}}\cdot\cos x$

$g''(0) = f(\sin 0)(-\sin 0) + f'(\sin 0)\cdot\cos 0\cdot\cos 0$

$= f(0)\cdot 0 + f'(0)\cdot 1$

$= 0 + (-3)\cdot 1 = \boxed{-3}$

d. If h(x)=xf(x), write the first four terms of the Taylor polynomial for h(x) about x=0.

$f(0) = 4$

$f'(0) = -3$

$f''(0) = 2$

$f^3(0) = -1$

$f(x) \approx 4 + (-3)(x) + \frac{2(x)^2}{2!} + \frac{(-1)(x)^3}{3!}$

$f(x) \approx 4 - 3x + x^2 - \frac{x^3}{6}$

h(x) = xf(x) so multiply f(x) by x

$h(x) \approx 4x - 3x^2 + x^3 - \frac{x^4}{6}$

Not a great question, but eh ---

e. Use Euler's method starting at x=2 with two steps of equal size to approximate f(4). Show the work that leads to your answer. For f, it is known the derivative is a function ONLY of x.

(x,y)	Δx	$\frac{dy}{dx}$	$\Delta y = \frac{dy}{dx}\Delta x$	$(x+\Delta x, y+\Delta y)$
$(2,2)$	1	1	1	$(3,3)$
$(3,3)$	1	2	2	$(4,5)$

$f(4) \approx 5$

f. If $j(x) = f'(x) + \frac{2}{x^2+5x+6}$, evaluate $\int_0^2 j(x)\,dx$.

$$\int_0^2 f'(x) + \frac{2}{x^2+5x+6}\,dx = \int_0^2 f'(x)\,dx + \int_0^2 \frac{2}{x^2+5x+6}\,dx = f(x)\Big|_0^2 + \int_0^2 \frac{2}{x^2+5x+6}\,dx =$$

$$f(2)-f(0)+\int_0^2 \frac{2}{x^2+5x+6}\,dx = -2+\int_0^2 \frac{2}{x^2+5x+6}\,dx \longrightarrow \text{Partial Fraction!}$$

$$= -2+\int_0^2 \frac{2}{x+2} - \frac{2}{x+3}\,dx = -2+\left[2\ln|x+2| - 2\ln|x+3|\right]\Big|_0^2$$

$$= -2+\left[2(\ln|x+2| - \ln|x+3|)\right]\Big|_0^2$$

$$= -2+\left[2\ln\left|\frac{x+2}{x+3}\right|\right]\Big|_0^2$$

$$\boxed{= -2+\left[2\ln\left|\frac{4}{5}\right| - 2\ln\left|\frac{2}{3}\right|\right]}$$

$$\frac{2}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3}$$

$(x+2)(x+3)$

$2 = A(x+3)+B(x+2)$

$x=-3 \rightarrow 2 = -B \rightarrow -2 = B$

$x=-2 \rightarrow 2 = A$