Feb Break MC — yes, you will have an entrance ticket (nonpartner!!) on these the Monday we return!

① $\boxed{B}$ $\int_{\pi/4}^{x} \cos(2t)\,dt$ $\quad u = 2t$, $du = 2dt$, $\frac{1}{2}du = dt$ $\quad t=x \;\; u=2x$, $t=\pi/4 \;\; u=\pi/2$ $\Rightarrow \frac{1}{2}\int_{\pi/2}^{2x} \cos(u)\,du = \frac{1}{2}\sin(u)\Big]_{\pi/2}^{2x} =$

$= \frac{1}{2}[\sin(2x) - \sin(\pi/2)] = \frac{1}{2}(\sin 2x - 1)$

② $\boxed{E}$ POI : y changes concavity
y' changes inc/dec
y'' changes signs

$y' = 3x^2 - 30x + 33$
$y'' = 6x - 30 = 0$ when $x = 5$

	4	$\boxed{x=5}$	6
y''	$-$		$+$

$33 \times 5 = 165$

POI @ $x=5$ $\quad y = 5^3 - 15(5^2) + 33(5) + 100 = 5\cdot5^2 - 15\cdot5^2 + 33(5) + 100$
$y = -10(5^2) + 33(5) + 100 = -250 + 165 + 100 = -150 + 165 = 15$

③ $\boxed{B}$ $3x^2 - 2xy + 3y = 1$

$x = 2$
$3\cdot4 - 2\cdot2y + 3y = 1$
$12 - 4y + 3y = 1$
$-y = -11$
$y = 11$

$6x - 2(x\frac{dy}{dx} + y\cdot1) + 3\frac{dy}{dx} = 0$
$6x - 2x\frac{dy}{dx} - 2y + 3\frac{dy}{dx} = 0$
$\frac{dy}{dx}(-2x + 3) = 2y - 6x$
$\frac{dy}{dx} = \frac{2y - 6x}{-2x + 3} = \frac{2(11) - 6(2)}{-2(2) + 3} = \frac{22 - 12}{-4 + 3} = \frac{10}{-1} = \boxed{-10}$

④ $\boxed{A}$ $\int_1^3 8x^{-3}\,dx = \frac{8x^{-2}}{-2}\Big]_1^3 = -\frac{4}{x^2}\Big]_1^3 = -4\left(\frac{1}{9} - \frac{1}{1}\right) = -4\left(\frac{1}{9} - \frac{9}{9}\right) = -4\left(\frac{-8}{9}\right) = \boxed{\frac{32}{9}}$

⑤ $\boxed{B}$ $\int_0^8 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^8 f(x)\,dx = \frac{1}{2}(2+5)(2) - \frac{1}{2}(3)(2) = 7 - 3 = \boxed{4}$

⑥ $\boxed{E}$ f is continuous, c is any point between a + b.
~~A~~ not true for any c
~~B~~ we don't know end values are equal
~~C~~ we don't know slope at x=c is zero
~~D~~ not true for any c
(E) $\lim_{x\to c} f(x) = f(c)$ definition of continuity!

7. $\boxed{D}$ $f(x) = x^2(3x+1)^{1/2}$

$$f' = x^2\left(\tfrac{1}{2}(3x+1)^{-1/2}(3)\right) + (3x+1)^{1/2}(2x)$$
$$f' = (3x+1)^{-1/2}\left[\tfrac{3}{2}x^2 + 2x(3x+1)\right]$$
$$f' = \frac{3/2\,x^2 + 6x^2 + 2x}{\sqrt{3x+1}} = \frac{3/2\,x^2 + \frac{12}{2}x^2 + 4/2\,x}{\sqrt{3x+1}}$$
$$f' = \frac{1}{2}\,\frac{(3x^2+12x^2+4x)}{\sqrt{3x+1}} = \frac{15x^2+4x}{2\sqrt{3x+1}}$$

8. $\boxed{D}$ $f(t) = \dfrac{t^3+t}{4t+1}$

$$f'(t) = \left.\frac{(4t+1)(3t^2+1)-(t^3+t)(4)}{(4t+1)^2}\right|_{t=-1}$$
$$f'(-1) = \frac{(-3)(4)-(-2)(4)}{(-3)^2} = \frac{-12+8}{9} = \frac{-4}{9}$$

9. $\boxed{A}$ $\displaystyle\int_2^{e+1} \frac{4}{x-1}\,dx = 4\ln|x-1|\Big]_2^{e+1} = 4(\ln(e+1-1)-\ln(2-1)) =$

$4(\ln(e)-\ln(1)) = 4(1-0)$

10. $\boxed{C}$ Total Distance $= \displaystyle\int_0^{16}|v(t)|\,dt = \int_0^{16} v(t)\,dt = \tfrac{1}{2}(4)(60)+(4)(30)+4(90)+\tfrac{1}{2}(4)(90)$

$= 120+120+360+180 = 780$

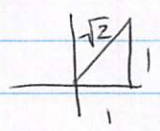
11. $\boxed{C}$ $\dfrac{d}{dx}\tan^2(4x) = \dfrac{d}{dx}\left[\tan(4x)\right]^2 = 2\tan(4x)\cdot\sec^2(4x)\cdot 4 =$

$8\tan(4x)\cdot\sec^2(4x)$

12. $\boxed{B}$ $y = \sin^2 x = (\sin x)^2$ at $x = \pi/4$ $\qquad y = (\sin \pi/4)^2 = (1/\sqrt{2})^2 = 1/2$

Tangent line: $y - y_0 = m(x - x_0)$

$$\boxed{y - 1/2 = 1(x - \pi/4)}$$



$m = y' = 2\sin x \cdot \cos x\Big|_{x=\pi/4} = 2(1/\sqrt{2})(1/\sqrt{2}) = 2(1/2) = 1$

13. $\boxed{B}$ f differentiable means (A) f continuous $\lim_{x\to1} f(x) = f(1)$

(B) f differentiable $\lim_{x\to1} f'(x)$ exists.

(A) f continuous: $\lim_{x\to1^-} f(x) = \overset{f(1)}{=} 3a + 2b + 1 = a - 4b - 3 = \lim_{x\to1^+} f(x)$

so $2a + 6b = -4$

(B) f differentiable: $\lim_{x\to1^-} f'(x) = 6ax + 2b = 4ax^3 - 8bx - 3 = \lim_{x\to1^+} f'(x)$

so $6a + 2b = 4a - 8b - 3$

from above: $2a + 10b = -3$ $\quad 2a = -3 - 10b$

$2a = -4 - 6b$ so $-4 - 6b = -3 - 10b$

$4b = 1$ $\quad b = 1/4$

14. $\boxed{A}$ $y = x^4 + 8x^3 - 72x^2 + 4$ concave down when $y'' < 0$

$y' = 4x^3 + 24x^2 - 144x$

$y'' = 12x^2 + 48x - 144 = 12(x^2 + 4x - 12) = 12(x+6)(x-2) = 0$

at $x = -6, x = 2$

	-7	$\boxed{-6}$	0	$\boxed{2}$	3
y''	$- - : +$		$+ - : -$		$+ + : +$

15. $\boxed{D}$ $\lim_{x\to-8} \frac{x^2+5x-24}{x^2+10x+16} = \lim_{x\to-8} \frac{\cancel{(x+8)}(x-3)}{\cancel{(x+8)}(x+2)} = \lim_{x\to-8} \frac{x-3}{x+2} = \frac{-11}{-6} = 11/6$

16. $\boxed{E}$ f(x) increasing $(-\infty,-2) \cup (2,\infty)$ so f' + there ~~A~~ ~~B~~ ~~C~~ ~~D~~

f(x) decreasing $(-2,2)$ so f' − there

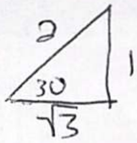
$f(x) \approx x^3 \quad f'(x) \approx 3x^2$

17. $\boxed{D}$ $f(x) = \ln(\cos(3x)) \quad f'(x) = \frac{1}{\cos(3x)} \cdot -\sin(3x) \cdot 3 = -3\frac{\sin(3x)}{\cos(3x)} =$

$\boxed{-3\tan(3x)}$

19. $\boxed{D}$ position: $x(t) = t^2 - 7t + 6$

velocity $v(t) = x'(t) = 2t - 7 = 0$ at $t = 7/2$



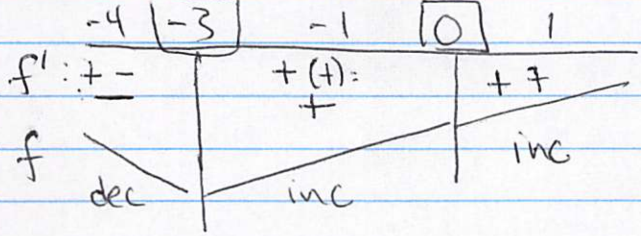
21. $\boxed{C}$ average value: $\frac{1}{\pi/4 - \pi/6}\int_{\pi/6}^{\pi/4} \sec^2 x\,dx = \frac{1}{\frac{3\pi}{12}-\frac{2\pi}{12}}\int_{\pi/6}^{\pi/4} \sec^2 x\,dx = \frac{1}{\pi/12}\int_{\pi/6}^{\pi/4}\sec^2 x\,dx$

$= \frac{12}{\pi}\cdot \tan x \Big]_{\pi/6}^{\pi/4} = \frac{12}{\pi}\left(\tan(\pi/4) - \tan(\pi/6)\right) = \frac{12}{\pi}\left(1-\frac{1}{\sqrt{3}}\right) = \frac{12}{\pi}\left(1-\frac{\sqrt{3}}{3}\right) =$

$\frac{12}{\pi}\left(\frac{3}{3}-\frac{\sqrt{3}}{3}\right) = \frac{12}{\pi}\left(\frac{3-\sqrt{3}}{3}\right) = \frac{4(3-\sqrt{3})}{\pi}$

23. $\boxed{A}$ $f(x) = x^4 + 4x^3$ f decreasing when $f'(x) < 0$

$f'(x) = 4x^3 + 12x^2 = 4x^2(x+3) = 0$ at $x=0$, $x=-3$



26. $\boxed{B}$ to find max velocity set derivative of velocity = 0

compare velocity at critical pts + end pts.

position: $s(t) = 2t^3 - 12t^2 + 16t + 2$

velocity: $v(t) = 6t^2 - 24t + 16$

acc: $a(t) = 12t - 24 = 0$ at $t = 2$ → $t=0$, $t=5$, $t=2$

$$\begin{array}{r} 16 \\ \times\ 6 \\ \hline 96 \end{array} \qquad \begin{array}{r} 24 \\ \times\ 4 \\ \hline 96 \end{array}$$

t	$v(t)$
0	$v(0) = 16$
2	$v(2) = 24 - 48 + 16 = -8$
5	$v(5) = 6(25) - 24(5) + 16 = 46$

27. $\boxed{E}$ Mean Value Thm: $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c on (a,b)

$(0,5)$

$\frac{f(5)-f(0)}{5-0} = \frac{125-30-0}{5} = \frac{95}{5} = 19$

take "+" one

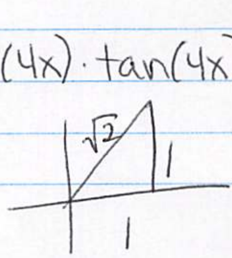
$f'(x) = 3x^2 - 6\big|_c = 3c^2 - 6 = 19$ so $3c^2 = 25$ $c^2 = \frac{25}{3}$

$c = \pm\sqrt{\frac{25}{3}} = \pm\frac{5}{\sqrt{3}}$

28. $\boxed{A}$ $f(x) = \sec(4x)$ $f'(x) = \sec(4x)\cdot\tan(4x)\cdot 4$

$f'(\pi/16) = \sec(\pi/4)\cdot\tan(\pi/4)\cdot 4$

$= \frac{\sqrt{2}}{1}\cdot(1)\cdot 4$



$\boxed{\text{Calc OK}}$

29 $\boxed{A}$ slope of tangent line: $f'(x) = e^{3x} \cdot 3 = 3e^{3x} = 2$?
graph $y_1 = 3e^{3x}$ $y_2 = 2$ find intersection @
$\boxed{x = -.135}$ window: $x: [-1, 1]$ $y: [0, 5]$

30. $\boxed{C}$ graph $y = x^3 + 12x^2 + 15x + 3$ window: $x: [-11, 0]$ $y: [-10, 200]$
2nd Trace Calc $\rightarrow$ Maximum @ $x = -7.317$

31. $\boxed{B}$ Given: $\frac{ds}{dt} = .4 \frac{cm}{sec}$ Area $= s^2$ $P = 4s = 2 \cdot 2s$
$\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$ so $2s = \frac{1}{2}P$
$\frac{dA}{dt} = \frac{1}{2}P \cdot (.4) = P(.2)$

32 $\boxed{D}$ $f(x) = 3^x$ slope of tangent line: $f'(x) = 3^x \cdot \ln(3) = 1$
graph $y_1 = 3^x \cdot \ln 3$ $y_2 = 1$ find intersection at
$x = -.0856$ window: $x: [-2, 2]$ $y: [0, 5]$

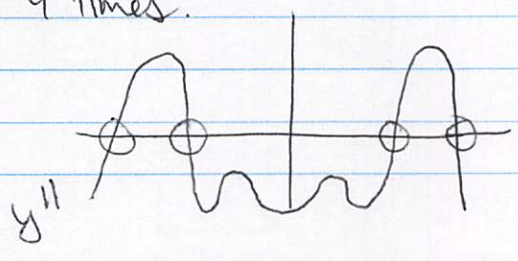
33. $\boxed{A}$ $h(x) = f(g(x))$ so $h'(x) = f'(g(x)) \cdot g'(x)$ so
$h'(a) = f'(g(a)) \cdot g'(a) = f'(c) \cdot b = 6 \cdot b$

35 $\boxed{C}$ $\frac{3-0}{4} = 3/4$ $[0, 3/4][3/4, 6/4][6/4, 9/4][9/4, 12/4]$

Trap: $\frac{1}{2}(e^0 + e^{3/4}) \cdot \frac{3}{4} + \frac{1}{2}(e^{3/4} + e^{6/4}) \cdot \frac{3}{4} + \frac{1}{2}(e^{6/4} + e^{9/4}) \cdot \frac{3}{4} + \frac{1}{2}(e^{9/4} + e^{12/4}) \cdot 3/4$

Trap: $\frac{1}{2}\left(\frac{3}{4}\right)\left[e^0 + 2e^{3/4} + 2e^{6/4} + 2e^{9/4} + e^{12/4}\right] = \frac{3}{8}[53.258) = 19.972$

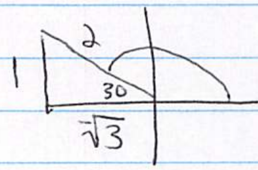
36. $\boxed{C}$ POI when y'' Δ's signs.
graph $y'' = x\sin x - 2$ on $(-10, 10)$ count # of times
y'' graph crosses x-axis 4 times!



37. $\boxed{E}$ $\lim\limits_{h\to 0} \dfrac{\sin\left(\frac{5\pi}{6}+h\right)-\frac{1}{2}}{h} = \lim\limits_{h\to 0} \dfrac{\sin\left(\frac{5\pi}{6}+h\right)-\sin\left(\frac{5\pi}{6}\right)}{h}$

this limit is saying "find the derivative of $\sin x$ at $x = 5\pi/6$.

$f = \sin x \quad f'(x) = \cos x \Big|_{5\pi/6} = -\cos\left(\frac{5\pi}{6}\right) = \boxed{\frac{-\sqrt{3}}{2}}$



(or Math 8
nDeriv ($\sin x$, x, $5\pi/6$) $= -.866$)

40. $\boxed{D}$ $y = x^3 + x^2$ at $y = 3$ graph $y_1 = x^3 + x^2$ $y_2 = 3$
intersection at $x = 1.175$

$m = y' = 3x^2 + 2x \Big|_{x=1.175} = 6.492$ pt $(1.175, 3)$

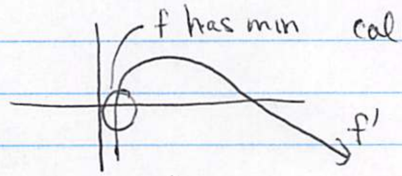
tangent line $y - 3 = 6.492(x - 1.175)$ $y = 6.492x - 7.62795 + 3$

41. $\boxed{D}$ f has relative minimum when $f'(x)$ changes from $-$ to $+$

f ∪

f′ $-$ $+$

graph $f'(x)$ on
$x: [-1, 5]$
$y: [-2, 2]$



calculate zero
$x = 0.159$

43. $\boxed{E}$ $f(x) = \int \cot x\,dx$ $f(1) = f(\pi/6) + \int_{\pi/6}^{1} \cot x\,dx = 1 + \underbrace{.5205}_{\text{math 9}}$

44. $\boxed{B}$ $\frac{dy}{dt} = ky$ so $y = C_1 e^{kt}$ ← t in seconds!

½ life 1 minute means
if $(0, C_1)$ $(60, \frac{1}{2}C_1)$

$\frac{1}{2}C_1 = C_1 e^{k(60)}$ (divide by C_1)

so $\frac{1}{2} = e^{60k}$ $\ln(\frac{1}{2}) = \ln e^{60k}$ so $\ln(\frac{1}{2}) = 60k$ $k = \frac{\ln(\frac{1}{2})}{60}$

$\boxed{k = -.01155}$

45. $\boxed{B}$ $g(2) = \int_0^2 f(x)\,dx \approx 4$

$g(4) = \int_0^4 f(x)\,dx \approx 4 - 3 = 1$

$g(6) = \int_0^6 f(x)\,dx \approx 4 - 3 + 2 = 3$

Integration Review

Partial Fractions

1. $\int \frac{x-9}{(x+5)(x-2)}\,dx \Rightarrow \int \frac{A}{x+5} + \frac{B}{x-2}\,dx \Rightarrow \int \frac{2}{x+5} - \frac{1}{x-2}\,dx = 2\ln|x+5| - \ln|x-2| + C$

$x-9 = A(x-2) + B(x+5)$

when $x=2 \longrightarrow B=-1$

when $x=-5 \rightarrow A=2$

2. $\int \frac{1}{(t+4)(t-1)}\,dt \Rightarrow \int \frac{A}{t+4} + \frac{B}{t-1}\,dt \Rightarrow \int \frac{-\frac{1}{5}}{t+4} + \frac{\frac{1}{5}}{t-1}\,dt = -\frac{1}{5}\ln|t+4| + \frac{1}{5}\ln|t-1| + C$

$1 = A(t-1) + B(t+4)$

when $t=1 \rightarrow B=\frac{1}{5}$

when $t=-4 \rightarrow A=-\frac{1}{5}$

3. $\int_2^3 \frac{1}{x^2-1}\,dx = \int_2^3 \frac{1}{(x-1)(x+1)}\,dx \Rightarrow \int_2^3 \frac{A}{x-1} + \frac{B}{x+1}\,dx \Rightarrow \int_2^3 \frac{\frac{1}{2}}{x-1} - \frac{\frac{1}{2}}{x+1}\,dx = \left[\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1|\right]\Big|_2^3$

$1 = A(x+1) + B(x-1)$

when $x=-1 \rightarrow B=-\frac{1}{2}$

when $x=1 \rightarrow A=\frac{1}{2}$

$= \left[\frac{1}{2}\ln(2) - \frac{1}{2}\ln(4)\right] - \left[\frac{1}{2}\cancel{\ln(1)} - \frac{1}{2}\ln(3)\right]$

$= \frac{1}{2}\ln\left(\frac{2}{4}\right) + \frac{1}{2}\ln(3) = \frac{1}{2}\ln\left(\frac{1}{2}\right) + \frac{1}{2}\ln(3) = \frac{1}{2}\ln\left(\frac{3}{2}\right)$

4. $\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \int_0^1 \frac{x-1}{(x+2)(x+1)}\,dx \Rightarrow \int_0^1 \frac{A}{x+2} + \frac{B}{x+1}\,dx \Rightarrow \int_0^1 \frac{3}{x+2} - \frac{2}{x+1}\,dx = \left[3\ln|x+2| - 2\ln|x+1|\right]\Big|_0^1$

$x-1 = A(x+1) + B(x+2)$

when $x=-1 \rightarrow B=-2$

when $x=-2 \rightarrow A=3$

$= \left[3\ln(3) - 2\ln(2)\right] - \left[3\ln(2) - 2\cancel{\ln(1)}\right]$

$= 3\ln(3) - 2\ln(2) - 3\ln(2) = 3\ln(3) - 5\ln(2)$

$= \ln(27) - \ln(32) = \ln\left(\frac{27}{32}\right)$

5. $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy \Rightarrow \int_1^2 \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3}\,dy \Rightarrow \int_1^2 \frac{2}{y} + \frac{\frac{9}{5}}{y+2} + \frac{\frac{1}{5}}{y-3}\,dy = \left[2\ln|y| + \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3|\right]\Big|_1^2$

$4y^2-7y-12 = A(y+2)(y-3) + B(y)(y-3) + C(y)(y+2)$

when $y=0 \rightarrow -12 = A(2)(-3) \rightarrow A=2$

when $y=-2 \rightarrow 18 = B(-2)(-5) \rightarrow B=\frac{9}{5}$

when $y=3 \rightarrow 3 = C(3)(5) \rightarrow C=\frac{1}{5}$

$= \left[2\ln(2) + \frac{9}{5}\ln(4) + \frac{1}{5}\cancel{\ln(1)}\right] - \left[2\cancel{\ln(1)} + \frac{9}{5}\ln(3) + \frac{1}{5}\ln(2)\right]$

$= 2\ln(2) + \frac{9}{5}\ln(4) - \frac{9}{5}\ln(3) - \frac{1}{5}\ln(2)$

$= \frac{9}{5}\ln(2) + \frac{9}{5}\ln(4) - \frac{9}{5}\ln(3) = \frac{9}{5}\ln\left(\frac{2\cdot 4}{3}\right)$

$= \frac{9}{5}\ln\left(\frac{8}{3}\right)$

6. $\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{x^2+2x-1}{x(x+1)(x-1)}\,dx \Rightarrow \int \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}\,dx \Rightarrow \int \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x-1}\,dx =$

$\ln|x| - \ln|x+1| + \ln|x-1| + C$

$x^2+2x-1 = A(x+1)(x-1) + B(x)(x-1) + C(x)(x+1)$

when $x=0 \rightarrow -1 = A(1)(-1) \rightarrow A=1$

when $x=-1 \rightarrow -2 = B(-1)(-2) \rightarrow B=-1$

when $x=1 \rightarrow 2 = C(1)(2) \rightarrow C=1$

AP Calculus Integration by Parts Worksheet

Evaluate the following by hand.

1) $\int x(x+1)^8\,dx$

$$u = x \qquad dv = (x+1)^8\,dx$$
$$du = dx \qquad v = \frac{1}{9}(x+1)^9$$
$$\frac{1}{9}x(x+1)^9 - \int \frac{1}{9}(x+1)^9\,dx$$
$$\frac{1}{9}x(x+1)^9 - \frac{1}{9}\left(\frac{1}{10}\right)(x+1)^{10} + C$$

2) $\int xe^{-x}dx$

$$u = x \qquad dv = e^{-x}dx$$
$$du = dx \qquad v = -e^{-x}$$
$$-xe^{-x} - \int -e^{-x}dx = -xe^{-x} - e^{-x} + C$$

3) $\int x\ln 2x\,dx$

$$u = \ln 2x \qquad dv = x\,dx$$
$$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$$
$$\frac{x^2\ln 2x}{2} - \int \frac{x}{2}dx = \frac{x^2\ln 2x}{2} - \frac{x^2}{4} + C$$

4) $\int x(\ln x)^2\,dx$

$$u = (\ln x)^2 \qquad dv = x\,dx$$
$$du = \frac{2\ln x}{x}dx \qquad v = \frac{x^2}{2}$$
$$\frac{x^2(\ln x)^2}{2} - \int x\ln x\,dx \qquad \begin{array}{ll} u = \ln x & dv = x\,dx \\ du = \frac{1}{x}dx & v = \frac{x^2}{2} \end{array}$$
$$\frac{x^2(\ln x)^2}{2} - \left[\frac{x^2}{2}\ln x - \int \frac{x}{2}dx\right] = \frac{x^2(\ln x)^2}{2} - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$$

5) $\int x^2\sin x\,dx$

$$u = x^2 \qquad dv = \sin x\,dx$$
$$du = 2x\,dx \qquad v = -\cos x$$
$$-x^2\cos x + \int 2x\cos x\,dx \qquad \begin{array}{ll} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{array}$$
$$-x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right] = -x^2\cos x + 2x\sin x + 2\cos x + C$$

6) $\int e^{2x} \sin x \, dx$

$$u = e^{2x} \qquad dv = \sin x \, dx$$
$$du = 2e^{2x} dx \qquad v = -\cos x$$
$$-e^{2x} \cos x + 2\int e^{2x} \cos x \, dx \qquad \begin{matrix} u = e^{2x} & dv = \cos x \, dx \\ du = 2e^{2x} dx & v = \sin x \end{matrix}$$
$$-e^{2x} \cos x + 2\left[e^{2x} \sin x - 2\int e^{2x} \sin x \, dx\right] = -e^{2x} \cos x + 2e^{2x} \sin x - 4\int e^{2x} \sin x \, dx$$

$$\int e^{2x} \sin x \, dx = \frac{1}{5}\left[-e^{2x} \cos x + 2e^{2x} \sin x\right] + C$$

7) $\int_0^{\pi} x \sin 2x \, dx$

$$u = x \qquad dv = \sin 2x \, dx$$
$$du = dx \qquad v = -\frac{1}{2}\cos 2x$$
$$\left[-\frac{1}{2} x \cos 2x\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} \cos 2x \, dx = \left[-\frac{1}{2} x \cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi} = -\frac{\pi}{2}$$

8) $\int_0^1 x^2 e^x \, dx$

$$u = x^2 \qquad dv = e^x dx$$
$$du = 2x \, dx \qquad v = e^x$$
$$x^2 e^x \Big|_0^1 - 2\int_0^1 x e^x dx \qquad \begin{matrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{matrix}$$
$$x^2 e^x \Big|_0^1 - 2\left[x e^x \Big|_0^1 - \int_0^1 e^x dx\right] = e - 2\left[e - e^x \Big|_0^1\right] = e - 2\left[e - (e - 1)\right] = e - 2$$