AP Calculus Multiple Choice

Name:__________

1. $\lim_{x \to \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 $\boxed{\text{(B) } -2}$ (C) 2 (D) 3 (E) nonexistent

$\lim_{x \to \infty} \frac{6x - 2x^2 - 3 + x}{x^2 + 3x - x - 3} = \lim_{x \to \infty} \frac{-2x^2 + 7x - 3}{x^2 + 2x - 3} \Rightarrow$ End Behavior Model : $\lim_{x \to \infty} \frac{-2x^2}{x^2} = -2$

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2. $\int \frac{1}{x^2} dx = \int x^{-2} dx = -x^{-1} + C = -\frac{1}{x} + C$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ $\boxed{\text{(D) } -x^{-1} + C}$ (E) $-2x^{-3} + C$

Product w/ chain rule

chain rule

3. If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$ $(x-1)\cdot 3(x^2+2)^2\cdot 2x+(1)(x^2+2)^3$

first deriv of second deriv of first Keep second

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $\boxed{(x^2+2)^2(7x^2-6x+2)}$

(E) $-3(x-1)(x^2+2)^2$

$$f'(x)=6x(x-1)(x^2+2)^2+(x^2+2)^3$$
$$=(x^2+2)^2[6x(x-1)+(x^2+2)]$$
$$=(x^2+2)^2[6x^2-6x+x^2+2]$$
$$=(x^2+2)^2[7x^2-6x+2]$$

Fast u-sub

4. $\int(\sin(2x)+\cos(2x))\,dx=$ $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

divide by deriv of inside

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $\boxed{-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C}$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim_{x\to 0}\frac{5x^4+8x^2}{3x^4-16x^2}$ is

$\boxed{\text{(A) } -\frac{1}{2}}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

$$\lim_{x\to 0}\frac{5x^4+8x^2}{3x^4-16x^2} = \lim_{x\to 0}\frac{x^2(5x^2+8)}{x^2(3x^2-16)} = \lim_{x\to 0}\frac{5x^2+8}{3x^2-16} = -\frac{1}{2}$$

$\downarrow$

$\frac{0}{0}$

$$f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

✓ I. f has a limit at $x=2$. $\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} x+2 = 4$

X II. f is continuous at $x=2$. 1. $f(2)$ exists; 1 2. $\lim_{x\to 2} f(x)$ exists; 4 3. $f(2)\neq\lim_{x\to 2} f(x)$ fails continuity

X III. f is differentiable at $x=2$. continuity fails, so differentiability fails

$\boxed{\text{(A) I only}}$

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

7. A particle moves along the x-axis with velocity given by $v(t)=3t^2+6t$ for time $t\geq 0$. If the particle is at position $x=2$ at time $t=0$, what is the position of the particle at $t=1$?

(A) 4 $\boxed{\text{(B) 6}}$ (C) 9 (D) 11 (E) 12

$P(1)=P(0)+\int_0^1 v(t)\,dt$

$P(1)=2+\int_0^1 3t^2+6t\,dt=2+\left(t^3+3t^2\right)\Big|_0^1=2+(1+3)-(0)=6$

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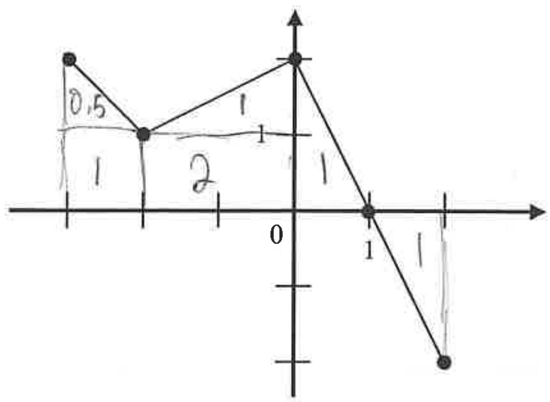
8. If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ $\boxed{\text{(E) } -\frac{3\sqrt{3}}{2}}$

$f(x)=\cos(3x)$

chain rule $f'(x)=-\sin(3x)\cdot 3$

$f'\left(\frac{\pi}{9}\right)=-\sin\left(3\cdot\frac{\pi}{9}\right)\cdot 3=-\sin\left(\frac{\pi}{3}\right)\cdot 3=-\frac{\sqrt{3}}{2}\cdot 3$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

Maximum

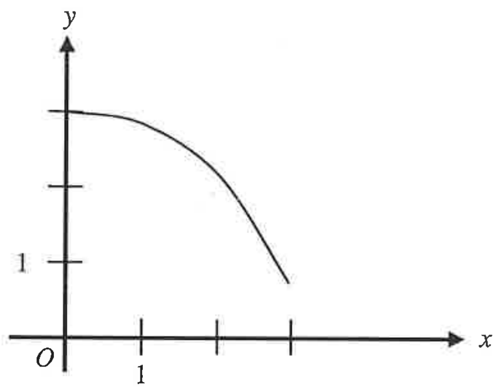
(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ $\boxed{\text{(D) } g(1)}$ (E) $g(2)$

Could find each value, but max is either at end points or critical points of g.

E.P. $x=-3$ $g(-3)=\int_{-2}^{-3} f(t)\,dt=-1.5$

$x=2$ $g(2)=\int_{-2}^{2} f(t)\,dt=2+1+1-1=3$

C.P. $x=1$ $g(1)=\int_{-2}^{1} f(t)\,dt=2+1+1=4$



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

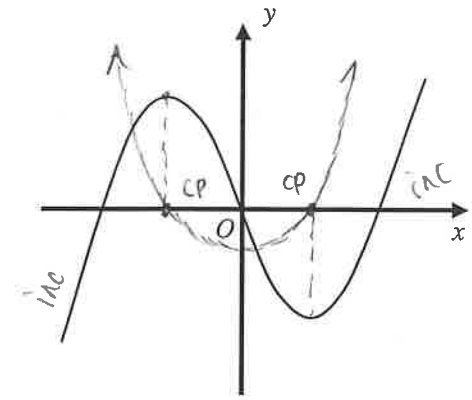
(A) $\int_1^3 f(x)\,dx$ → Exact value of area

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length over approx

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length under approx

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length splits the difference

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length under approx but closer than Right Riemann

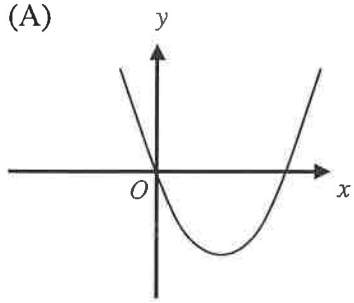


Graph of f

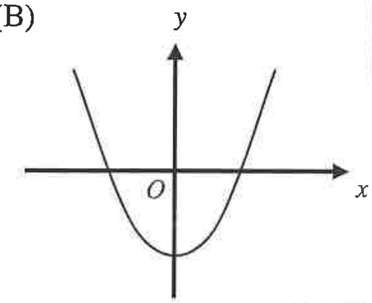
slope of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

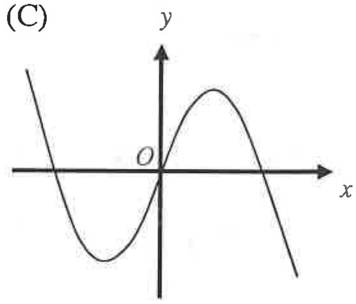
(A)



(B)



(C)



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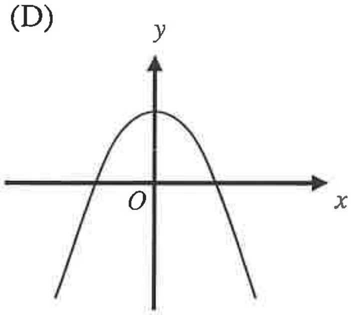
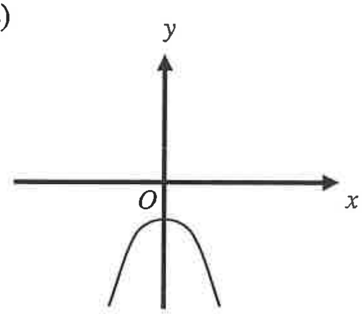


(E)



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ $\boxed{\text{(D) } -\frac{2}{x^2}e^{(2/x)}}$ (E) $-2x^2 e^{(2/x)}$

$f(x) = e^{2/x} = e^{2x^{-1}}$

chain rule $f'(x) = e^{2x^{-1}} \cdot (-2x^{-2}) = e^{2/x} \cdot \left(\frac{-2}{x^2}\right)$

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chain rule

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) = f'(\ln(x)) \cdot \frac{1}{x}$

$= [2(\ln(x)) + 2] \cdot \frac{1}{x}$

$\boxed{\text{(A) } \frac{2\ln x + 2}{x}}$ (B) $2x \ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

OR $f(\ln(x)) = [\ln(x)]^2 + 2\ln(x)$

$\frac{d}{dx} f(\ln(x)) = 2\ln(x) \cdot \frac{1}{x} + 2\frac{1}{x}$

x	0	1	2	3
$f''(x)$	5	0	−7	4

tells us about concavity

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

~~(A)~~ f is increasing on the interval $(0, 2)$. → f' question

~~(B)~~ f is decreasing on the interval $(0, 2)$. → f' question

~~(C)~~ f has a local maximum at $x = 1$. → f' question

~~(D)~~ The graph of f has a point of inflection at $x = 1$. could be true, but at $x=1$ we can't say for sure if f'' changes sign.

f'' changes sign

$\boxed{\text{(E)}}$ The graph of f changes concavity in the interval $(0, 2)$.

f'' changes sign: true somewhere on $(0,2)$.

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15. $\int \frac{x}{x^2-4}\,dx = \int \frac{x}{u} \cdot \frac{1}{2x}\,du = \int \frac{1}{2} \cdot \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2-4| + C$

$u = x^2 - 4$

$\frac{du}{dx} = 2x$

$\frac{1}{2x}\,du = dx$

(A) $\frac{-1}{4(x^2-4)^2} + C$

(B) $\frac{1}{2(x^2-4)} + C$

$\boxed{\text{(C) } \frac{1}{2}\ln|x^2-4| + C}$

(D) $2\ln|x^2-4| + C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

Implicit differentiation: x's and y's together

$\sin(xy) = x$

$\cos(xy) \cdot [\underbrace{x \frac{dy}{dx} + 1 \cdot y}_{\text{product rule for } x \cdot y}] = 1$

chain rule

$\cos(xy) x \frac{dy}{dx} + y\cos(xy) = 1$

$\cos(xy) x \frac{dy}{dx} = 1 - y\cos(xy)$

$\frac{dy}{dx} = \frac{1 - y\cos(xy)}{\cos(xy) x}$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

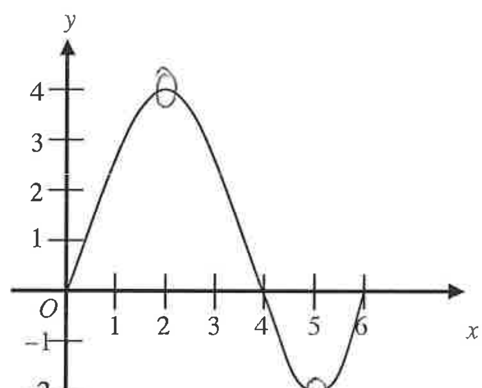
(D) $\boxed{\frac{1-y\cos(xy)}{x\cos(xy)}}$

(E) $\frac{y(1-\cos(xy))}{x}$

$g(x) = \int_0^x f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

g has POI when $g''(x) = f'(x)$ changes sign [where f changes inc → dec or vice versa].

(A) 2 only (B) 4 only $\boxed{\text{(C) 2 and 5 only}}$ (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

$\boxed{\text{(A) } -3}$ (B) -2 (C) -1 (D) 0 (E) 1

$X+Y=K$ is tangent to $y=x^2+3x+1$

$Y=K-X$

$Y=-X+K$

Slope of tangent line is -1

$y'=2x+3$

$-1=2x+3$

$-4=2x$

$x=-2$

when $x=-2$

$Y=(-2)^2+3(-2)+1$

$Y=4-6+1=-1$

$(-2,-1)$

tangent line goes through $(-2,-1)$

$-1=-(-2)+K$

$-1=2+K$

$-3=K$

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two limits

19. What are all horizontal asymptotes of the graph of $y = \dfrac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

$\boxed{\text{(E) } y = -1 \text{ and } y = 5}$

limits:

$\lim_{x\to\infty} \frac{5+2^x}{1-2^x} = \lim_{x\to\infty} \frac{2^x}{-2^x} = -1$

$\lim_{x\to-\infty} \frac{5+2^x}{1-2^x} = \lim_{x\to-\infty} \frac{5+0}{1-0} = 5$

negative exponents means $2^x \to 2^{-\infty} \Rightarrow \frac{1}{2^\infty} \Rightarrow 0$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

where $f''(x)$ changes sign

(A) 0 only

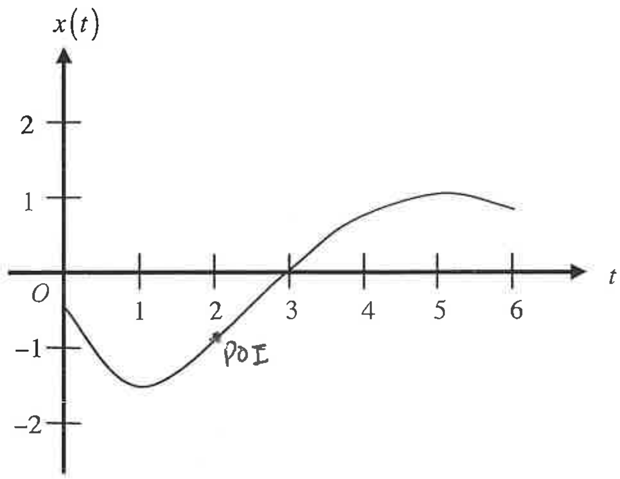
(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6

f''	$+$	0 $+$	$\boxed{3}$ $-$	$\boxed{6}$ $+$
x^2	$+$	$+$	$+$	$+$
$x-3$	$-$	$-$	$+$	$+$
$x-6$	$-$	$-$	$-$	$+$



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

↳ when v' is +
↳ v' is $a(t)$
↳ $a(t)$ is $x''(t)$
so when $x''(t)$ is +
↳ this means $x(t)$ is CCU

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

heard Not heard

$$\frac{dp}{dt} = kp(N-p)$$

(A) $\frac{dp}{dt} = kp$

(B) $\boxed{\frac{dp}{dt} = kp(N-p)}$

(C) $\frac{dp}{dt} = kp(p-N)$

(D) $\frac{dp}{dt} = kt(N-t)$

(E) $\frac{dp}{dt} = kt(t-N)$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y}$ with the initial condition $y(3)=-2$?

(A) $y=2e^{-9+x^3/3}$

(B) $y=-2e^{-9+x^3/3}$

(C) $y=\sqrt{\frac{2x^3}{3}}$

(D) $y=\sqrt{\frac{2x^3}{3}-14}$

$\boxed{\text{(E) } y=-\sqrt{\frac{2x^3}{3}-14}}$

$\frac{dy}{dx}=\frac{x^2}{y}$

$\int y\,dy=\int x^2\,dx$

$\frac{1}{2}y^2=\frac{1}{3}x^3+C$ $\quad (3,-2)$

$\frac{1}{2}(4)=\frac{1}{3}(27)+C$

$2=9+C$

$-7=C$

$\frac{1}{2}y^2=\frac{1}{3}x^3-7$

$y^2=\frac{2}{3}x^3-14$

$y=\pm\sqrt{\frac{2}{3}x^3-14}$

(3,-2) is the initial condition

$\therefore\ y=-\sqrt{\frac{2}{3}x^3-14}$

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24. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4 $\quad\boxed{\text{(B) 0.6}}\quad$ (C) 0.7 $\quad$ (D) 1.3 $\quad$ (E) 1.4

point: (2,1)
slope: 4

$y-1=4(x-2)$

$y=4(x-2)+1$ $\quad$ when $x=1.9$

$y=4(-0.1)+1=0.6$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

continuous and one-sided derivatives are continuous.

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) −4 $\quad$ $\boxed{\text{(B) } -2}$ $\quad$ (C) 0 $\quad$ (D) 2 $\quad$ (E) 4

Continuity

$cx+d = x^2 - cx$ at $x=2$

$2c+d = 4-2c$

$4c+d = 4$

Differentiability: derivatives

$c = 2x - c$ at $x=2$

$c = 4 - c$

$2c = 4$

$c = 2$

$4(2)+d = 4$

$d = -4$

$c+d = -2$

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26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

$\boxed{\text{(A) } 2}$ $\quad$ (B) $\frac{1}{2}$ $\quad$ (C) 0 $\quad$ (D) $-\frac{1}{2}$ $\quad$ (E) −2

Oops! unnecessary

Point: $y = \arctan(4 \cdot \frac{1}{4}) = \arctan(1) = \frac{\pi}{4}$

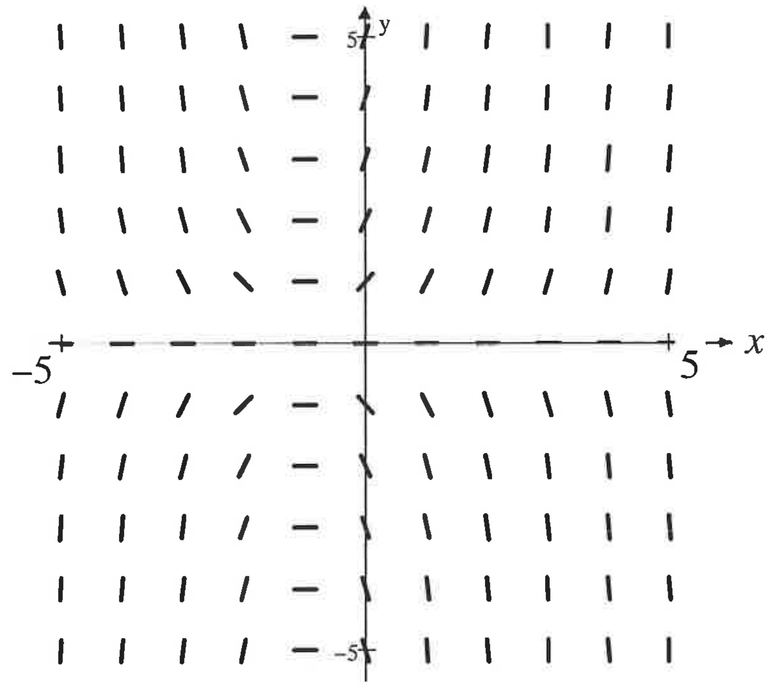
$x = \frac{1}{4}$

$(\frac{1}{4}, \frac{\pi}{4})$

slope: $y' = \frac{1}{1+(4x)^2} \cdot 4 = \frac{4}{1+16x^2}$

when $x = \frac{1}{4}$

$y' = \frac{4}{1+16(\frac{1}{4})^2} = \frac{4}{2} = 2$



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$ slope is 0 when $x=-1$ or $y=0$
zero when x or $y=0$

zeroes

(B) $\frac{dy}{dx} = xy - y = y(x-1)$ $y=0,\ x=1$

(C) $\boxed{\frac{dy}{dx} = xy + y = y(x+1)}$ $y=0,\ x=-1$

(D) $\frac{dy}{dx} = xy + x = x(y+1)$ $y=-1,\ x=0$

(E) $\frac{dy}{dx} = (x+1)^3$ $x=-1$

28. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

$$g'(3)=\frac{d}{dx}f^{-1}(3)=\frac{1}{f'(f^{-1}(3))}=\frac{1}{f'(6)}=\frac{1}{-2}$$

$$f^{-1}(3)=6$$

$\boxed{\text{(A) } -\frac{1}{2}}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.