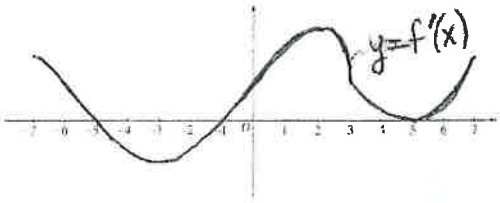
Name:
AP Calculus BC

Unit 5 Review Problems

1) The graph of f'(x), the derivative of f is shown at right. Identify the x-values on[-7,7] (and justify them!) where:



a. f(x) achieves relative/local maxima:
b. f(x) has a point of inflection:
c. f(x) is increasing
d. f(x) is concave down
e. f(x) is decreasing and concave up
f. f(x) is increasing and concave down

a) f(x) has a local max when f'(x) changes from positive to negative: at x=-5
b) f(x) has a point of infleetion when f'(x) changes from increasing to decreasing or vice versa: at x=-3, 2, 5
c) f(x) is increasing when f'(x) is positive: [-7,-5) and (-1,5) and (5,7]
d) f(x) is concave down when f'(x) is decreasing: (-7,-3) and (2,5)
e) f(x) is decreasing and concave up when f'(x) is negative and increasing: (-3,-1)
f) f(x) is increasing and concave down when f'(x) is positive and decreasing: (-7,-5) and (2,5)

5. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is decreasing?

f(x) is decreasing when f'(x) is negative.

$f'(x) = 3x^2 + 6x - 9$

$f'(x) = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$

CP: $x = -3, 1$

	(+)	-3	(−)	1	(+)
3	+		+		+
(x+3)	−		+		+
(x−1)	−		−		+

f(x) is decreasing on (-3, 1)

3. If $f''(x) = e^x(x-2)(x+3)^3$, then at what x-values does the graph of f have inflection points?

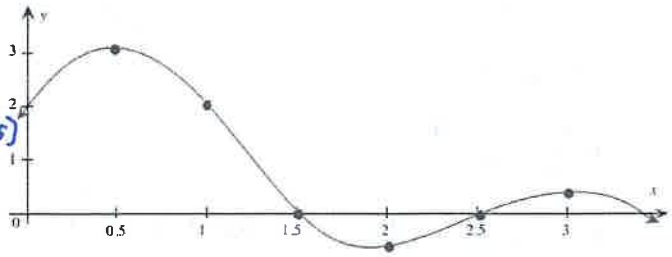
f has inflection points when f''(x) changes sign.

	(+)	-3	(−)	2	(+)
e^x	+		+		+
(x-2)	−		−		+
$(x+3)^3$	−		+		+

POI at x = -3, 2

4. The graph of f''(x) is shown at the right.

f''(x)



assume domain [0, 3.5]

a. At what x-values does f(x) have points of inflection?

f(x) has points of inflection when f''(x) changes sign: at x = 1.5 and 2.5

b. On what intervals is the graph of f(x) concave up?

f(x) is CCU when f''(x) is positive: on [0, 1.5) and (2.5, 3.5)

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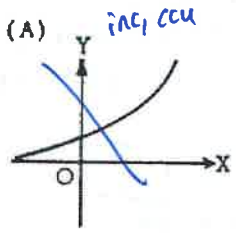
Critical Point max ∩ CCD min ∪ CCU

2. Suppose $f(x)$ is a function which satisfies $f'(3) = 0$, $f'(5) = 0$, $f''(3) = -4$, and $f''(5) = 5$. Which of the following statements is true?

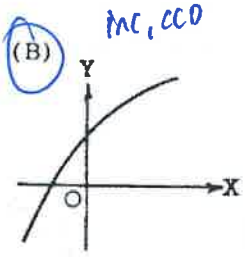
(A) $f(x)$ has a relative minimum at $x = 3$ and at $x = 5$.
(B) $f(x)$ has a relative minimum at $x = 3$ and a relative maximum at $x = 5$.
(C) $f(x)$ has a relative maximum at $x = 3$ and at $x = 5$.
(D) $f(x)$ has a relative maximum at $x = 3$ and a relative minimum at $x = 5$. ✓
(E) None of the above is true.

↗ increasing ↗ CCD

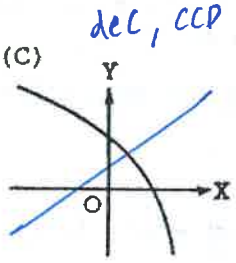
16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

(A) 

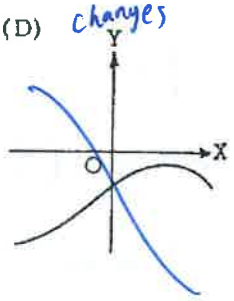
inc, ccu

(B) 

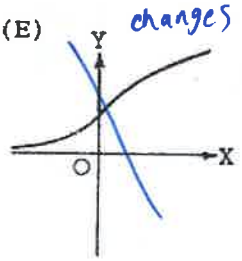
inc, ccd

(C) 

dec, ccd

(D) 

changes

(E) 

changes

2nd deriv changes sign.

17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) (0,0) only (B) (3,162) only (C) (4,256) only
(D) (0,0) and (3,162) (E) (0,0) and (4,256)

$y' = 20x^3 - 5x^4$

$y'' = 60x^2 - 20x^3 = 20x^2(3 - x)$

zeroes: $x = 0$ and $x = 3$
↳ won't change sign because x^2 ↳ POI

⇒

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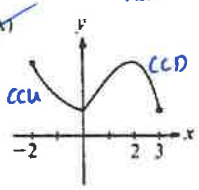
f not differentiable at $x=0$.

43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f?

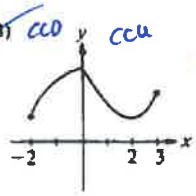
CP, horiz tan

(A)



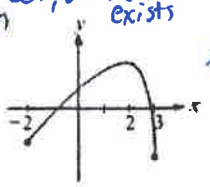
CCD

(B)

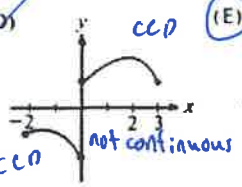


CCD, but f'(0) exists

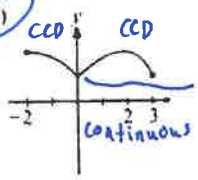
(C)



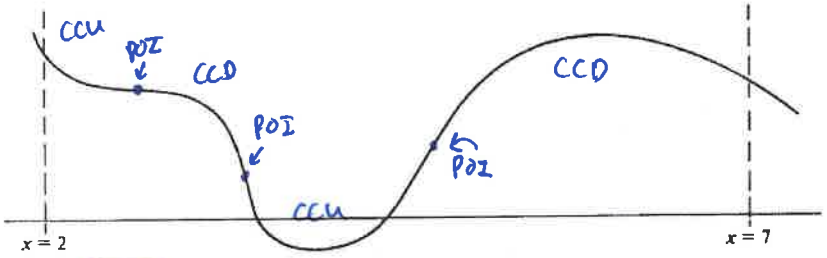
(D)



(E)



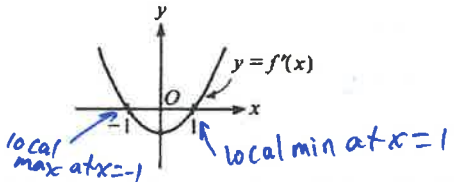
f'(0) Does not exist



20. The graph of $y=f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

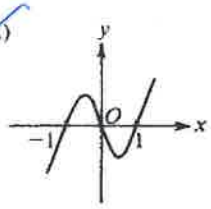
where f(x) changes concavity type.

(A) One (B) Two (C) Three (D) Four (E) Five

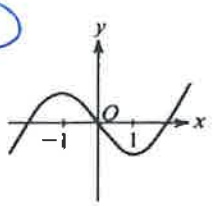


33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

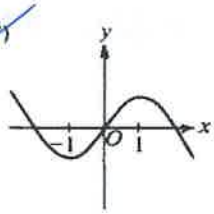
(A)



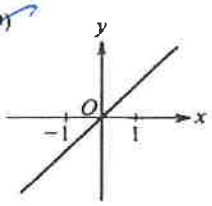
(B)



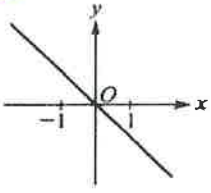
(C)



(D)



(E)



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Calculator Ok.

5. If $f'(x) = 3\sin(\frac{1}{2}x^2 - 5x + 1)$ for 0<x<4, on what intervals is f(x) concave down? Justify.

f(x) is concave down when f'(x) is decreasing: this is on $(0, 0.544)$ and $(1.315, 2.299)$

Calculator Ok.

6. For the function above, at what x-values does f(x) have relative maxima? Justify.

f(x) has relative maxima when f'(x) changes from positive to negative: this is at $x = 0.204$ and $x = 1.769$

No Calculator

7. The absolute minimum value of $F(x) = x^3 - 6x^2 - 1$ on the closed interval $[-1, 5]$ occurs at x=

(a) -1 (b) 0 (c) 2 (d) 4 (e) 5

Absolute min could be at CPs or EPs

EPs:
$x = -1$
$x = 5$

$F'(x) = 3x^2 - 12x = 3x(x-4)$

CPs: $x = 0$
$x = 4$

$F(-1) = -8$
$F(5) = -26$

$F(0) = -1$
$F(4) = -33$