AP Calculus BC
Name:

Integration Technique Practice

Basic Antidifferentiation

1. $\int 5t^8 - 2t^4 + t + 3 \; dt =$ $\frac{5}{9}t^9 - \frac{2}{5}t^5 + \frac{1}{2}t^2 + 3t + C$

2. $\int 5\sqrt{y} - \frac{3}{\sqrt{y}} \; dy =$ $\int 5y^{1/2} - \frac{3}{y^{1/2}} dy = \int 5y^{1/2} - 3y^{-1/2} dy = \frac{10}{3}y^{3/2} - 6y^{1/2} + C$

3. $\int 7\cos(x) + 4e^x \; dx =$ $7\sin(x) + 4e^x + C$

Fast U-Substitution

4. $\int \sin(4x) \, dx =$ $-\frac{1}{4}\cos(4x) + C$

5. $\int \frac{1}{7w} dw =$ $\frac{1}{7}\ln|7w| + C$

U-Substitution

6. $\int 5\sec^2(5x+1) \, dx =$ $\int 5\sec^2(u) \frac{1}{5} du = \int \sec^2(u) du = \tan(5x+1) + C$

$u = 5x+1$

$\frac{du}{dx} = 5$

$\frac{1}{5} du = dx$

7. $\int_3^5 \frac{2x-3}{\sqrt{x^2-3x+1}} dx =$ $\int_1^{11} \frac{2x-3}{\sqrt{u}} \frac{1}{2x-3} du = = \int_1^{11} u^{-1/2} du = 2u^{1/2}\Big|_1^{11} = 2\sqrt{11} - 2\sqrt{1} = 2\sqrt{11} - 2$

$u = x^2 - 3x + 1$

$\frac{du}{dx} = 2x - 3$

$\frac{1}{2x-3} du = dx$

$x = 5 \rightarrow u = 11$

$x = 3 \rightarrow u = 1$

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$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = 4$

Integration Using Completing the Square

8. $\int \frac{1}{5r^2-20r+100} dr = \int \frac{1}{5(r^2-4r+20)} dr = \frac{1}{5}\int \frac{1}{r^2-4r+20} dr = \frac{1}{5}\int \frac{1}{(r^2-4r+4)+20-4} dr$

$= \frac{1}{5}\int \frac{1}{(r-2)^2+16} dr = \frac{1}{5}\cdot\frac{1}{4}\tan^{-1}\left(\frac{r-2}{4}\right)+C$

Note: $\int \frac{1}{u^2+a^2} du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$

Integration Using Polynomial Long Division

9. $\int \frac{2x+1}{x+2} dx = \int 2 + \frac{-3}{x+2} dx = \int 2\,dx - \int \frac{3}{x+2} dx = 2x - 3\ln|x+2| + C$

$$\begin{array}{r} 2 \\ x+2 \overline{)\,2x+1} \\ -(2x+4) \\ \hline -3 \end{array}$$

Integration Using Partial Fraction Decomposition

10. $\int \frac{6x+13}{x^2+5x+6} dx = \int \frac{A}{x+3} + \frac{B}{x+2} dx = \int \frac{5}{x+3} + \frac{1}{x+2} dx = 5\ln|x+3| + \ln|x+2| + C$

$6x+13 = A(x+2) + B(x+3)$

$x=-2$ $\quad B=1$

$x=-3$ $\quad A=5$

Integration by Parts

11. $\int \sqrt{x}\ln(x)\,dx = \ln(x)\cdot\frac{2}{3}x^{3/2} - \int \frac{2}{3}x^{3/2}\cdot\frac{1}{x} dx = \frac{2}{3}\ln(x)\sqrt{x^3} - \int \frac{2}{3}x^{1/2} dx$

$= \frac{2}{3}\ln(x)\sqrt{x^3} - \frac{4}{9}x^{3/2} + C$

$u = \ln(x)$ $\quad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}dx$ $\quad v = \frac{2}{3}x^{3/2}$

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will take repeated Int by Parts

12. $\int x^2\cos(3x)\,dx = x^2\cdot\frac{1}{3}\sin(3x) + 2x\cdot\frac{1}{9}\cos(3x) - \frac{2}{27}\sin(3x) + C$

$\frac{d}{dx} f(x)$		$\int g(x)$
x^2	$+$	$\cos(3x)$
$2x$	$-$	$\frac{1}{3}\sin(3x)$
2	$+$	$-\frac{1}{9}\cos(3x)$
0		$-\frac{1}{27}\sin(3x)$

Improper Integrals

13. $\int_0^{\infty} xe^{-x^2}\,dx = \lim\limits_{a\to\infty}\int_0^a xe^{-x^2}\,dx = \lim\limits_{a\to\infty}\int_0^{-a^2} xe^{u}\left(-\frac{1}{2x}\right)du = \lim\limits_{a\to\infty}\int_0^{-a^2} -\frac{1}{2}e^{u}\,du$

$u = -x^2$

$\frac{du}{dx} = -2x$

$\frac{1}{-2x}\,du = dx$

$x = a \to u = -a^2$

$x = 0 \to u = 0$

$= \lim\limits_{a\to\infty}\left(-\frac{1}{2}e^{u}\right)\Big|_0^{-a^2} = \lim\limits_{a\to\infty}\left(-\frac{1}{2}e^{-a^2}\right) - \left(-\frac{1}{2}e^{0}\right)$

$= \lim\limits_{a\to\infty}\left(\frac{-1}{2e^{a^2}}\right) + \frac{1}{2} = \frac{1}{2}$

14. $\int_0^1 \frac{1}{x}\,dx = \lim\limits_{a\to 0^+}\int_a^1 \frac{1}{x}\,dx = \lim\limits_{a\to 0^+}\left(\ln|x|\right)\Big|_a^1 = \lim\limits_{a\to 0^+}\left(\underset{\to 0}{\ln(1)} - \underset{\to -\infty}{\ln(a)}\right) \Rightarrow \boxed{\text{Diverges}}$