Intervention 1: Limits and Continuity

Definition of a Limit:	
A function is continuous if:	
What can make a function fail to be continuous?	Horizontal Asymptotes:

3. $\lim_{x\to 3} \dfrac{x-3}{x^2-2x-3}$ is

(A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

7. For what value of k does $\lim_{x\to 4} \dfrac{x^2-x+k}{x-4}$ exist?

A) -12

B) -4

C) 3

D) 7

E) No such value exists.

Test Strategy:

Let f be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

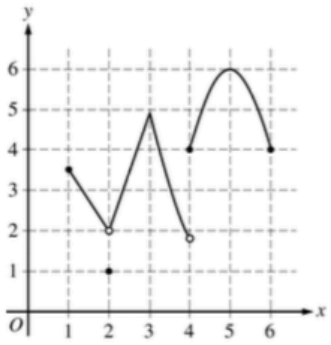
(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim\limits_{x\to 2} f(x)$ exists.

(B) $\lim\limits_{x\to 3} f(x)$ exists.

(C) $\lim\limits_{x\to 4} f(x)$ exists.

(D) $\lim\limits_{x\to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

For which of the following does $\lim\limits_{x\to\infty} f(x) = 0$?

I. $f(x) = \dfrac{\ln x}{x^{99}}$

II. $f(x) = \dfrac{e^x}{\ln x}$

III. $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x-5}$ (D) $y = \dfrac{5x}{1-x}$ (E) $y = \dfrac{20x^2 - x}{1+4x^2}$

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.

20. Let f be the function given by $f(x) = \dfrac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f, then $b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

86. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$

89. If $\lim_{x \to a} f(x) = f(a)$, then which of the following statements about f must be true?

(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.

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6. What are all the horizontal asymptotes of $f(x) = \dfrac{6 + 3e^x}{3 - e^x}$ in the xy-plane?

A. $y = 3$ only B. $y = -3$ only C. $y = 2$ only D. $y = -3$ and $y = 0$ E. $y = -3$ and $y = 2$

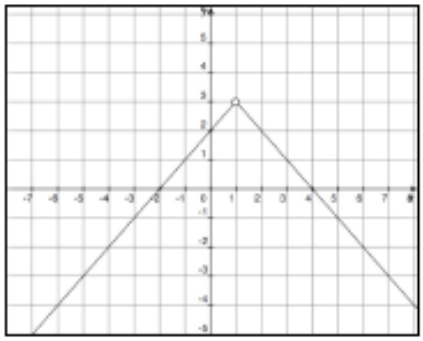
$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f?

I. $\lim_{x \to 3} f(x)$ exists
II. f is continuous at $x = 3$.
III. f is differentiable at $x = 3$.

a. None
b. I only
c. II only
d. I and II only
e. I, II, and III

3. The figure to the right shows the graph of $f(x)$.
Which of the following statements are true?



I. $\lim_{x \to 1^-} f(x)$ exists

II. $\lim_{x \to 1^+} f(x)$ exists

III. $\lim_{x \to 1} f(x)$ exists

A. I only B. II only C. I and II only D. I, II and III E. none are true