I.2 Definition of Derivative and Calculus Theora

1. Suppose we only know a few values of the function f(x).

f(x)	5	7	10	4	3	-2	4	6	7
x	0	2	3	6	8	9	11	12	14

a. What is the average rate of change of f(x) over the interval [2,11]?

b. What is the slope of the secant line through the points f(3) and f(8)?

c. Approximate the instantaneous rate of change of f(x) at x=7.

Average Rate of Change:	Instantaneous Rate of Change:

Definition of a Derivative

$$\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{3}+h\right)-\sin\left(\frac{\pi}{3}\right)}{h} \text{ is}$$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

3. Find: $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-\sin\left(\frac{\pi}{2}\right)}{h}$

4. Find: $\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2}+h\right)}{h}$

$$\lim_{x\to 2}\frac{\ln(x+3)-\ln(5)}{x-2} \text{ is}$$

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

5. Find $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}}{h}$

6. Find $\lim_{x\to \frac{\pi}{4}}\frac{\sin x-\sin\frac{\pi}{4}}{x-\frac{\pi}{4}}$

What is the average rate of change of $y=\cos(2x)$ on the interval $\left[0,\frac{\pi}{2}\right]$?

(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

IVT, MVT, and EVT

Intermediate Value Theorem: Criteria: What it proves: What you need to show:	Mean Value Theorem: Criteria: What it proves: What you need to show:
Extreme Value Theorem: Criteria: What it proves: What you need to show:	

Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.