I.6 Improper Integrals

Improper integrals are integrals that have infinity as a limit of integration OR that contain a vertical asymptote (infinite discontinuity) on their interval of integration.

Examples: Evaluate each integral or show that it is divergent.

$$\int_1^\infty \frac{6}{3x-1}\,dx$$

$$\int_1^4 \frac{1}{(2x-5)^2}\,dx$$

Practice: Evaluate the integral or show that it is divergent.

$$\int_1^\infty \frac{1}{(3x+1)^2}\,dx$$

BC 2004 Form B #5 No Calc

Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

(a) Find the average value of g on the closed interval $[1, 4]$.

(d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be

$\lim_{b \to \infty} \left[\frac{\int_a^b f(x)\,dx}{b - a} \right]$. Show that the improper integral $\int_4^\infty g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

BC 2017 #5 No Calc

5. Let f be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

(b) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\int_5^\infty f(x)\,dx$ or show that the integral diverges.

(d) Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.