I.8 Series and Convergence Tests

What do you remember are the important ideas in each of the following topics?

Geometric Series	p-Series
Alternating Series Test	Ratio Test

Some Practice

1. For what values of x do the following series converge? This is the radius of convergence.
   a. $\sum_{n=0}^{\infty} 2^n x^n$

   b. $\sum_{n=0}^{\infty} (-1)^n (x+1)^n$

2. Are the series below absolutely convergent, conditionally convergent, or divergent?
   a. $\sum_{n=1}^{\infty} \frac{(-1)^n n^{3/2}}{n^6}$

   b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

3. For what values of x do the following series converge? This is the radius of convergence.
   a. $\sum_{n=0}^{\infty} n(3x-5)^n$

   b. $\sum_{n=0}^{\infty} \frac{n(x-1)^n}{5^n}$

Intervention Practice Questions

Name

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{5^n \cdot n^p}$, where p is a constant and $p > 0$.

(a) For $p = 3$ and $x = 8$, does the series converge absolutely, converge conditionally, or diverge? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

(b) For $p = 1$ and $x = 8$, does the series converge absolutely, converge conditionally, or diverge? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

(c) When $x = -2$, for what values of p does the series converge? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Intervention Practice Questions

(d) When $p = 1$ and $x = 3.1$, the series converges to a value S. Use the first two terms of the series to approximate S. Use the alternating series error bound to show that this approximation differs from S by less than $\frac{1}{300{,}000}$.

Please respond on separate paper, following directions from your teacher.

Let $a_n = \dfrac{1}{n \ln n}$ for $n \geq 3$.

2. Consider the infinite series $\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \dfrac{1}{3 \ln 3} - \dfrac{1}{4 \ln 4} + \dfrac{1}{5 \ln 5} - \cdots$. Identify the properties of this series that guarantee the series coverage. Explain why the sum of this series is less than $\dfrac{1}{3}$.

Please respond on separate paper, following directions from your teacher.

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3. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Intervention Practice Questions

The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{nx^n}{2n^2+1}$ and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

(a) Use the ratio test to find R.

Please respond on separate paper, following directions from your teacher.

(b) Determine whether the series converges absolutely, converges conditionally, or diverges when $x = R$. Justify your answer.

Please respond on separate paper, following directions from your teacher.

(c) Determine whether the series converges absolutely, converges conditionally, or diverges when $x = -R$. Justify your answer.

Please respond on separate paper, following directions from your teacher.

(d) The first ten terms of the Maclaurin series for f are used to approximate $f(-1)$. Show that this approximation differs from $f(-1)$ by less than $\frac{1}{10}$.

Please respond on separate paper, following directions from your teacher.