Logs, Exponents, and Trig Review

If…	It means…	Or…
The particle is to the right of the origin.	Position is positive	Position graph is above the t-axis.
The particle is to the left of the origin.	Position is negative	Position graph is below the t-axis.
The particle is at the origin.	Position is 0.	Position graph is at the t-axis.
The particle is moving to the left.	Velocity is negative.	Velocity graph is below the t-axis.
The particle is moving to the right.	Velocity is positive.	Velocity graph is above the t-axis.
The particle is at rest.	Velocity is 0.	Velocity graph is at the t-axis.
The acceleration of the particle is positive.	Velocity is increasing.	Velocity graph is "going up"
The acceleration of the particle is ~~positive~~. negative	Velocity is decreasing.	Velocity graph is "going down".

1. A population p can be modeled by the function $P = Ce^{kt}$ where C and k are both constants. If the population triples every 5 years, what is are the two constants?

t, P
$(0,1) \to (5,3)$

$1 = Ce^{k(0)}$
$1 = Ce^{0}$
$1 = C$

$3 = e^{k(5)}$
$\ln(3) = k \cdot 5$
$\frac{\ln(3)}{5} = k$

$P = e^{\frac{\ln(3)}{5}t}$

2. The decay of a specific isotope can be modeled by the function $A = Ce^{-kt}$ where C and k are both constants. If the amount of the isotope halves every 15 years, find a specific model for A, the amount of the isotope.

$t\ A$
$(0,2) \longrightarrow (15,1)$

$A = Ce^{-k(0)}$
$2 = Ce^{0}$
$C = 2$

$1 = 2e^{-k(15)}$
$\frac{1}{2} = e^{-k15}$
$\ln(\frac{1}{2}) = -15k$
$\frac{\ln(\frac{1}{2})}{-15} = k$

$A = 2e^{-\frac{\ln(\frac{1}{2})}{\cancel{-15}}t}$
$\boxed{A = 2e^{\frac{\ln(\frac{1}{2})}{15}t}}$

32. $\log_3 7 - \log_3 x = \log_3(7/x)$

33. $2\log_5 x + \log_5 3 = \log_5(x^2) + \log_5(3) = \log_5(x^2 \cdot 3)$

34. $\log_4 5 + \log_4 x + \log_4 y = \log_4(5xy)$

35. $3\log_{10} x - \log_{10} 4 = \log_{10}(x^3) - \log_{10}(4) = \log_{10}\left(\frac{x^3}{4}\right)$

36. $2\log_2 x - 3\log_2 y = \log_2(x^2) - \log_2(y^3)$
$\log_2\left(\frac{x^2}{y^3}\right)$

37. $\log_3 4 + 2\log_3 x - \log_3 5 = \log_3(4) + \log_3(x^2) - \log_3(5)$
$= \log_3(4x^2/5)$

38. $\log_2 x - 2\log_s y = \log_2(x) - \log(y^2)$
$\log_2(x/y^2)$

39. $3\log_a 2 + \log_a 6 - 2\log_a 4$
$\log_a(8) + \log_a(6) - \log_a(16)$
$\log_a(48/16) = \log_a(3)$

Evaluate the following Trig Expressions. Rationalize every denominator.

$\sec(\pi/6) = \frac{2\sqrt{3}}{3}$

$\csc(\pi/4) = \sqrt{2}$

$\cot(\pi/3) = \frac{\sqrt{3}}{3}$

$\sec(\pi/3) = 2$

$\csc(\pi/6) = 2$

$\cot(\pi/4) = 1$

$\sec^2(0) = 1$

$\csc^2(\pi/2) = 1$

$\cot^2(0) = \text{undef}$

1. A. If $y = \sin\left(\frac{\pi}{4}x\right)$, find the zeroes on the interval $[0, 2\pi]$.

$0 = \sin\left(\frac{\pi}{4}x\right)$

$\sin^{-1}(0) = \frac{\pi}{4}x$

$\frac{\pi}{4}x = 0 + 2\pi n$

$x = \frac{4}{\pi}(0 + 2\pi n)$

$x = 0 + 8n$

$x = 8n$

$\frac{\pi}{4}x = \pi + 2\pi n$

$x = 4 + 8n$

$n = 0$: $\boxed{x = 0}$ $\boxed{x = 4}$

$n = 1$: $\cancel{x = 8}$ $\cancel{x = 12}$

B. Use your graphing calculator to confirm your answer.

2. A. If $y = \cos\left(\frac{\pi}{5}x\right)$, find the zeroes on the interval $[0, 2\pi]$.

$\rightarrow 6.28$

$0 = \cos\left(\frac{\pi}{5}x\right)$

$\cos^{-1}(0) = \frac{\pi}{5}x$

$\frac{\pi}{5}x = \frac{\pi}{2} + 2\pi n$

$x = \frac{5}{2} + 10n$

$n=0$

$\boxed{x = \frac{5}{2}}$

$\frac{\pi}{5}x = \frac{3\pi}{2} + 2\pi n$

$x = \frac{15}{2} + 10n$

$\cancel{x = \frac{15}{2}}$ too big

B. Use your graphing calculator to confirm your answer.

3. There are two particles moving along the x-axis. One particle moves with a velocity given by the following equation: $v_1(t) = e^{\frac{3}{2}t} - 4$. The other particle moves with a velocity given by the equation $v_2(t) = e^{\frac{1}{2}t} - 4$.

a. When do each of the particles stop moving?

$v_1 = e^{\frac{3}{2}t} - 4 = 0$

$e^{\frac{3}{2}t} = 4$

$\frac{3}{2}t = \ln(4)$

$\boxed{t = \frac{2\ln(4)}{3}}$

$v_2 = e^{\frac{1}{2}t} - 4 = 0$

$e^{\frac{1}{2}t} = 4$

$\frac{1}{2}t = \ln(4)$

$\boxed{t = 2\ln(4)}$