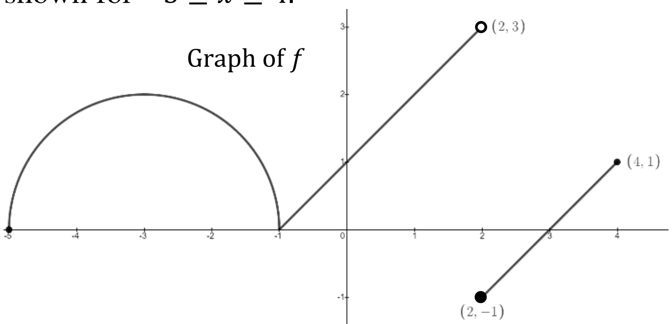
Unit 1: Limits & Continuity The piecewise function $f(x)$ is made of two line segments and a semi-circle as shown for $-5 \le x \le 4$.



a) $\lim_{x\to 0} f(x-1) = \lim_{x\to -1} f(x) = 0$

b) $\lim_{x\to 0} f(2-x^2) = \lim_{u\to 2^-} f(u) = 3$

c) $\lim_{x\to 0} \left(\frac{|x|}{x} \cdot f(x-1)\right) = 0$

$\lim_{x\to 0+} \left(\frac{|x|}{x} \cdot f(x-1)\right) = 1 \cdot 0 = 0;\quad \lim_{x\to 0^-} \left(\frac{|x|}{x} \cdot f(x-1)\right) = -1 \cdot 0 = 0$

Unit 2: Differentiation

Given $f(-3) = 5, f'(-3) = -2$, and $g(x) = \frac{1}{x}$. Let $h(x) = 4f(x)\cdot g(x)$. Find $\lim_{x\to -3} \frac{h(x)-h(-3)}{x+3} = h'(-3)$.

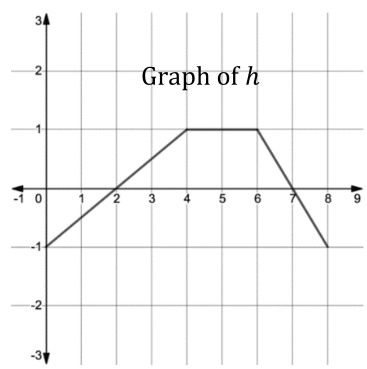
$$h'(x) = 4f'(x)g(x) + 4f(x)g'(x)$$

$$h'(-3) = 4f'(-3)g(-3) + 4f(-3)g'(-3) = 4(-2)\left(-\frac{1}{3}\right) + 4(5)\left(-\frac{1}{9}\right) = \frac{4}{9}$$

Unit 3: Chain Rule

1.

x	$g(x)$	$g'(x)$
1	2	0.2
2	4	0.4
3	5	0.6
4	8	0.8



The function g is differentiable. The table gives values of g and its derivative g' at selected values of x. The function h, whose graph is shown above, consists of three line segments.

(a) Let k be the function defined by $k(x) = h(g(x))$. Find $k'(1)$.

$$k'(x) = h'(g(x))g'(x)$$

$$k'(1) = h'(g(1))g'(1) = h'(2)g'(1) = \frac{1}{2}(0.2) = \frac{1}{10}$$

(b) Let m be the function defined by $m(g(x)) = x$. In other words, m and g are inverses. Find $m'(4)$.

$$m'(4) = \frac{1}{g'(m(4))} = \frac{1}{g'(2)} = \frac{1}{0.4} = \frac{5}{2}$$

2. Given the differential equation $\frac{dW}{dt} = \frac{1}{10}(W-600)$, find $\frac{d^2W}{dt^2}$ in terms of W.

$$\frac{d^2W}{dt^2} = \frac{1}{10}\cdot\frac{dW}{dt} = \frac{1}{10}\left(\frac{1}{10}(W-600)\right)$$

Unit 4: Contextual Applications of the Derivative

At the beginning of 2020, a landfill contained 1500 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{10}(W - 600)$ for the next 10 years. W is measured in tons, and t is measured in years from the start of 2020.

Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of April 2020 (time $t = \frac{1}{3}$).

$$W(0) = 1500; \qquad m = \left.\frac{dW}{dt}\right|_{W=1500} = \frac{1}{10}(1500 - 600) = 90$$

$$y = 1500 + 90(x - 0)$$

$$W\left(\frac{1}{3}\right) \approx 1500 + 90\left(\frac{1}{3}\right) = 1530$$

Unit 5: Analytical Applications of the Derivative

1. At the beginning of 2020, a landfill contained 1500 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{10}(W - 600)$ for the next 10 years. W is measured in tons, and t is measured in years from the start of 2020. Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of April 2020 (time $t = \frac{1}{3}$). Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine if the tangent line approximation is an underestimate or an overestimate of the solid waste that the landfill contains at the time $t = \frac{1}{3}$.

$$\frac{d^2W}{dt^2} = \frac{1}{10}\left(\frac{1}{10}(W - 600)\right) = \frac{1}{100}W - 6$$

For all $t > 0, W > 1500$. So $\frac{d^2W}{dt^2} > 0$ for all $t > 0$.

Thus the tangent line approximation is an underestimate.

2. Verify that the function $g(x) = \sqrt{x+2}$ satisfies the hypotheses of the Mean Value Theorem (MVT) on the interval $[-2, 0]$. Find all number(s) $x = c$, $-2 < c < 0$, that satisfy the conclusion of the MVT.

$g(x)$ is continuous on $[-2, 0]$ and differentiable on $(-2, 0)$.

Therefore there is a value c such that $g'(c) = \frac{g(-2) - g(0)}{-2 - 0} = \frac{0 - \sqrt{2}}{-2}$

$$\frac{1}{2\sqrt{c+2}} = \frac{\sqrt{2}}{2}$$

$$\sqrt{c+2} = \frac{1}{\sqrt{2}}$$

$$c + 2 = \frac{1}{2}$$

$$c = -\frac{3}{2}$$

3. If $f(1) = 3$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, then what is the least value for $f(4)$?

Minimum value of $f'(x)$ is 2, so $m \geq 2$. So $f(4) \leq f(1) + 2(4 - 1) = 3 + 2(3) = 9$

Unit 6: Integration and Accumulation of Change

Rewrite as a definite integral and evaluate: $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3\cos\left(\frac{\pi k}{n}\right)+5\right)\left(\frac{\pi}{n}\right)$.

$$\Delta x = \frac{\pi}{n} = \frac{b-a}{n}\text{; width of interval is } \pi.$$

There is a direct operation on $\frac{\pi k}{n}$, so $x_k = 0 + \frac{\pi k}{n}$; the left bound is 0 and the right bound is π.

Function is $3\cos(x) + 5$.

$$\int_0^{\pi}(3\cos x + 5)\,dx = (3\sin x + 5x)\Big|_0^{\pi} = 3\sin\pi + 5\pi - \left(3\sin 0 + 5(0)\right) = 5\pi$$

Unit 7: Differential Equations

LORDY we have done enough of these!! Watch episodes 1-8, 21, 22 and search shared folder for resources.