AP Calculus BC

Power and Taylor Series Practice Problems

Part 1: Multiple Choice

1. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(x+\frac{3}{2}\right)^n$ converges?

   (A) $-\frac{5}{2} < x < -\frac{1}{2}$ **(B) $-\frac{5}{2} < x \leq -\frac{1}{2}$** (C) $-\frac{5}{2} \leq x < -\frac{1}{2}$ (D) $-\frac{1}{2} < x < \frac{1}{2}$ (E) $x < -\frac{1}{2}$

2. Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$ ?

   (A) $1 - x + x^2 - x^3 + \cdots$

   (B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

   **(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$**

   (D) $1 + x^2 + x^4 + x^6 + \cdots$

   (E) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

3. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function $f$ about $x = 0$. What is the value of $f^{(4)}(0)$ ?

   (A) 0 (B) $\frac{1}{4}$ (C) 6 (D) 24 **(E) 144**

4. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$ ?

   (A) $1 < x < 5$
   **(B) $1 \leq x < 5$**
   (C) $1 \leq x \leq 5$
   (D) $2 < x < 4$
   (E) $2 \leq x \leq 4$

5. What is the coefficient of $x^2$ in the Taylor series for $sin^2 x$ about $= 0$ ?

   (A) -2 (B) -1 (C) 0 **(D) 1** (E) 2

6. The coefficient of $\left(x-\frac{\pi}{4}\right)^3$ in the Taylor series about $\frac{\pi}{4}$ of $f(x) = cosx$ is

(A) $\frac{\sqrt{3}}{12}$ (B) $-\frac{1}{12}$ (C) $\frac{1}{12}$ **(D) $\frac{1}{6\sqrt{2}}$** (E) $-\frac{1}{3\sqrt{2}}$

7. The nth derivative of a function f at x = 0 is given by $f^{(n)}(0) = (-1)^{n+1}\frac{n+2}{3^n(n+1)}$ for all n $\geq$ 0. Which of the following is the Maclaurin series of f?

(A) $2-\frac{x}{2}+\frac{4x^2}{27}-\frac{5x^3}{1088}+\cdots$

(B) $-2+\frac{x}{2}-\frac{4x^2}{27}+\frac{5x^3}{108}+\cdots$

(C) $2+\frac{x}{2}-\frac{2x^2}{27}+\frac{5x^3}{648}+\cdots$

(D) $-2+\frac{3x}{2}-\frac{2x^2}{27}+\frac{5x^3}{648}+\ldots$

**(E) $-2+\frac{x}{2}-\frac{2x^2}{27}+\frac{5x^3}{648}+\cdots$**

8. A function *f* has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$
Which of the following is an expression for $f(x)$?

(A) $-3x\sin(x)+3x^2$

(B) $-\cos(x)+x^2$

(C) $-\cos(x^2)+1$

**(D) $x^2e^x-x^3-x^2$**

(E) $e^{x^2}-x^2-1$

9. What is the sum of the series $1+\ln 2+\frac{(\ln 2)^2}{2!}+\cdots+\frac{(\ln 2)^n}{n!}+\cdots$?

(A) $\ln 2$

(B) $\ln(1+\ln 2)$

**(C) 2**

(D) $e^2$

(E) The series diverges.

Part 2: Short Response

10. The Taylor series about $x = 5$ for a certain function *f* converging to *f(x)* for all *x* in the interval of convergence. The *nth* derivative of *f* at *x* = 5 is given by $f^{(n)}(5) = \frac{(-1)^n n!}{2^n(n+2)}$ and $f(5) = \frac{1}{2}$

Use the LaGrange error to show that the fourth-degree Taylor Polynomial for *f* about *x* = 5 approximates *f(6)* with an error less than $\frac{1}{200}$.

$$\boldsymbol{R_4 = \left|\frac{f^5(z)}{5!}(x-c)^5\right| = \left|\frac{5!}{2^5(7)5!}\right| = \frac{1}{224} < \frac{1}{200}}$$

Part 3: Free Response Questions

11. The function g is continuous for all real numbers *x* and is defined by

$g(x) = \frac{\cos(2x)-1}{x^2}$ for $x \neq 0$

(A) Let $f$ be the function given by $f(x) = \cos(2x)$. Write the first four nonzero terms and the general term of the Taylor series for $f$ about $x = 0$.

$$\boldsymbol{1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots + \frac{(-1)^n(2x)^{2n}}{(2n)!} + \cdots}$$

(B) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for $g$ about $x = 0$.

$$\boldsymbol{-\frac{2^2}{2!} + \frac{2^4x^2}{4!} - \frac{2^6x^4}{6!} + \cdots + \frac{(-1)^n 2^{2n} x^{2n-2}}{(2n)!} + \cdots}$$

(C) Determine whether $g$ has a relative maximum, relative minimum, or neither at $x = 0$. Justify your answer.

$\boldsymbol{g'(0) = 0 \; and \; g''(0) = \frac{4}{3} > 0}$**, therefore** $\boldsymbol{g(x)}$ **has a relative minimum at** $\boldsymbol{x = 0}$

12. The Maclaurin series for $e^x$ is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function $f$ is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function $f$ has derivatives of all orders at $x = 1$.

(A) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about x = 1.

$$\mathbf{1 + (x-1)^2 + \frac{(x-1)^4}{2!} + \frac{(x-1)^6}{3!} + \cdots + \frac{(x-1)^{2n}}{n!} + \cdots}$$

(B) Use the Taylor series found in part (a) to write the first four nonzero terms of the general term of the Taylor series for $f$ about x = 1.

$$\mathbf{1 + \frac{(x-1)^2}{2!} + \frac{(x-1)^4}{4!} + \frac{(x-1)^6}{6!} + \cdots + \frac{(x-1)^{2n-2}}{n!} \quad or \quad \frac{(x-1)^{2n}}{(n+1)!}}$$

(C) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

$$\mathbf{IOC\ (-\infty, \infty)}$$

13. The Maclaurin series for $\ln(\frac{1}{1-x})$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \leq x < 1$.

(A) Find the Maclaurin series for $\ln(\frac{1}{1+3x})$ and determine the interval of convergence.

$$\mathbf{\sum_{n=1}^{\infty} \frac{(-3x)^n}{n} \qquad IOC\ \left(-\frac{1}{3}, \frac{1}{3}\right]}$$

(B) Find the value $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$$\mathbf{\ln \frac{1}{2}}$$