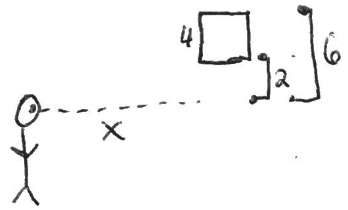
Problem Set

1. A 4ft x 4ft picture hangs on a wall such that its bottom edge is 2 ft above your eye level. How far back from the picture should you stand. directly in front of the picture, in order to view the picture under the maximum angle?

$x = \sqrt{2 \cdot 6} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3} \approx 3.464\text{ ft}$



OR

$\text{Angle} = \tan^{-1}\left(\frac{6}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$

Max angle when: $x = 3.464\text{ ft}$

2. A particle is moving along a line at a velocity of $y = 3\sin\left(\frac{2\pi x}{5}\right) + 2\,\frac{\text{m}}{\text{s}}$ at location x meters from the starting point on the line for $0 \le x \le 20$.
   a. Find a formula that represents the location of the particle given its velocity.
   b. What is the domain and range of the function you found in part (a)?
   c. Use your answer to part (a) to find where the particle is when it is traveling $5\,\frac{\text{m}}{\text{s}}$ for the first time.
   d. How can you find the other locations the particle is traveling at this speed?

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a. **Find a formula that represents the location of the particle given its velocity.**

$$y = \frac{5}{2\pi} \cdot \sin^{-1}\left(\frac{x-2}{3}\right)$$

b. **What is the domain and range of the function you found in part (a)?**

The domain is $-1 \le x \le 5$, and the range is $-\frac{5}{4} \le y \le \frac{5}{4}$.

c. **Use your answer to part (a) to find where the particle is when it is traveling $5\,\frac{\text{m}}{\text{s}}$ for the first time.**

The particle will be located at $x = 1.2$ meters from the starting point on the line.

d. **How can you find the other locations the particle is traveling at this speed?**

In this case, the velocity is a maximum, so it will only occur once every period. All other values can be found by adding multiples of 5 to the location. If it was not a maximum, we could subtract the location from $\frac{5}{2}$ to find another value within the same period and then add multiples of 5 to find analogous values in other periods.

6.15 PS

3. At a particular harbor over the course of 24 hours, the following data on peak water levels was collected (measurements are in feet above the MLLW):

Time	$1:30$	$7:30$	$14:30$	$20:30$
Water Level	-0.211	8.21	-0.619	7.518

a. What appears to be the average period of the water level?

b. What appears to be the average amplitude of the water level?

c. What appears to be the average midline for the water level?

d. Fit a curve of the form $y = A\sin(\omega(x-h)) + k$ or $y = A\cos(\omega(x-h)) + k$ modeling the water level in feet as a function of the time.

e. According to your function, how many times per day will the water level reach its maximum?

f. How can you find other time values for a particular water level after finding one value from your function?

g. Find the inverse function associated with the function in part (d). What is the domain and range of this function? What type of values does this function output?

At a particular harbor over the course of 24 hours, the following data on peak water levels was collected (measurements are in feet above the MLLW):

Time	1:30	7:30	14:30	20:30
Water Level	-0.211	8.21	-0.619	7.518

a. **What appears to be the average period of the water level?**

It takes 13 hours to get from the first low-point to the second, and 13 hours to get from the first high-point to the second, so the average period is $\frac{13+13}{2} = 13$.

b. **What appears to be the average amplitude of the water level?**

There are three areas we can examine to get amplitudes, from 1:30 to 7:30, 7:30 to 14:30, and 14:30 to 20:30. We get amplitudes of $\frac{8.21-(-0.211)}{2} = 4.2105$, $\frac{8.21-(-0.619)}{2} = 4.4145$, *and* $\frac{7.518-(-0.619)}{2} = 4.0685$. *We get* 4.231 *as the average amplitude.*

c. **What appears to be the average midline for the water level?**

3.748

d. **Fit a curve of the form $y = A\sin(\omega(x-h)) + k$ or $y = A\cos(\omega(x-h)) + k$ modeling the water level in feet as a function of the time.**

Since it would make our curve more inaccurate to guess at what point the water levels will cross the midline, we can either start at 1:30 or 7:30 and use the cosine function. For 7:30, we get

$$y = 4.231\cos\left(\frac{2\pi}{13}(x-7.5)\right) + 3.748.$$

e. **According to your function, how many times per day will the water level reach its maximum?**

It should reach its maximum levels twice a day usually, but there is the rare possibility that it will reach its maximum only once.

f. **How can you find other time values for a particular water level after finding one value from your function?**

The values repeat every 13 hours, so immediately once a value is found, adding any multiple of 13 will give other values that work. Additionally, if you have the inverse cosine value for a particular water level (but have not solved for x yet), then take the opposite, solve normally, and you will have another time to which you can add multiples of 13.

g. **Find the inverse function associated with the function in part (d). What is the domain and range of this function? What type of values does this function output?**

$$y = \frac{13}{2\pi}\cos^{-1}\left(\frac{x-3.748}{4.231}\right)+7.5$$

The domain is all real numbers x, such that $-0.483 \leq x \leq 7.979$, and the range is all real numbers y such that $7.5 \leq y \leq 14$.

Use the law of Sines and Cosines.

15. Find the x to the nearest whole number.

$$\frac{25}{\sin(148)} = \frac{12}{\sin(Z)}$$

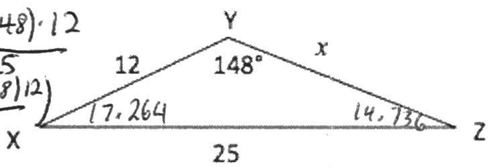
$$\sin(Z) = \frac{\sin(148)\cdot 12}{25}$$

$$\angle Z = \sin^{-1}\left(\frac{\sin(148)12}{25}\right)$$

$\angle Z = 14.736^\circ$

That means

$\angle X = 17.264$

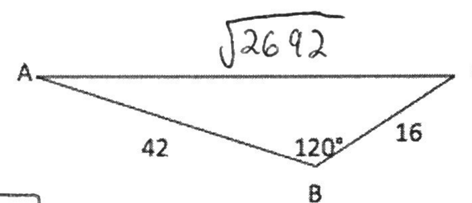


$$\frac{25}{\sin(148)} = \frac{x}{\sin(17.264)}$$

$x = 14.001$

14

16. Find the m$\angle A$ to the nearest whole degree.



$b^2 = 42^2 + 16^2 - 2(42)(16)\cos(120)$

$b = \sqrt{2692}$

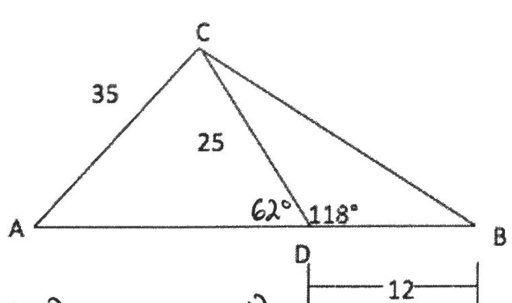
$$\frac{\sin(120)}{\sqrt{2692}} = \frac{\sin(A)}{16}$$

$$\sin^{-1}\left(\frac{16\sin(120)}{\sqrt{2692}}\right) = A = 15.490$$

15°

IV. Challenge Problems

17. Find the m$\angle A$ to the nearest whole degree.



$$\frac{\sin(62)}{35} = \frac{\sin(A)}{25}$$

$$\sin^{-1}\left(\frac{25\sin(62)}{35}\right) = A = 39.100^\circ$$

39°