Quarter 1 AP Calc Review

1. $\boxed{B}$ $f' < 0 \Leftrightarrow f$ decreasing $\quad f'' < 0 \Leftrightarrow f$ concave down

2. $\boxed{E}$ tangent $\Leftrightarrow$ same slope $\quad 2x(2y\frac{dy}{dx}) + y^2 \cdot 2 - 3\frac{dy}{dx} = 0$

plug in $(3,2)$ $\quad (2)(3)(2)(2)\frac{dy}{dx} + (2)^2(2) - 3\frac{dy}{dx} = 0$

$24\frac{dy}{dx} + 8 - 3\frac{dy}{dx} = 0$

$21\frac{dy}{dx} = -8 \quad \frac{dy}{dx} = -8/21$

3. $\boxed{C}$ pt of inflection means f changes concavity $\Leftrightarrow f''$ changes sign

$f(x) = x^{-1} + x^{1/2}$

$f'(x) = -1x^{-2} + \frac{1}{2}x^{-1/2}$

$f''(x) = 2x^{-3} - \frac{1}{4}x^{-3/2} = \frac{2}{x^3} - \frac{1}{4x^{3/2}} = 0 \quad \frac{2}{x^3} = \frac{1}{4x^{3/2}}$

only answer choice available must be.

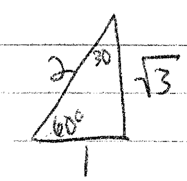
$f''(x)$ UD @ $x=0$

$8x^{3/2} = x^3$ (cube root) so $2x^{1/2} = x$ (squared) so $4x = x^2 \Rightarrow x = 4$

Sign chart to check

	−1	0	1	4	5
f''	$-2-\frac{1}{4}$ −		$2-\frac{1}{4}$ +		$\frac{2}{125} - \frac{1}{4(25)}$ = −
f	concave ↓		concave ↑		concave ↓

4. $\boxed{D}$ LT: $\frac{-4x^3}{8x^3} \sim \frac{-4}{8} \sim \frac{-1}{2}$



5. $\boxed{A}$ $f(x) = \cos(3\pi x^2)$

$f'(x) = -\sin(3\pi x^2) \cdot (2 \cdot 3\pi x)$

$f'(\frac{1}{3}) = -\sin\left(3\pi(\frac{1}{3})^2\right) \cdot (6\pi(\frac{1}{3})) = -\sin(\frac{\pi}{3}) \cdot 2\pi = -\frac{\sqrt{3}}{2} \cdot 2\pi = \boxed{-\sqrt{3}\pi}$

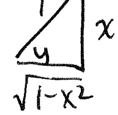
6. $\boxed{D}$ I not true (f has a corner @ $x=2$ so $f'(2)$ undefined)

$\boxed{II}$ true function values line up (no jumps, discontinuities)

$\boxed{III}$ $f' < 0$ means f decreasing on $(-1, 2)$ True

$y = \sin^{-1}x$

$x = \sin y$



take derivative: $1 = \cos y \frac{dy}{dx}$

$\frac{1}{\cos y} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}\Big|_{x=}$

7. $\boxed{D}$ $f(x) = \sin^{-1}(x)$ $\quad f'(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad f^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}}$

$= \frac{1}{\sqrt{\frac{4}{4}-\frac{2}{4}}} = \frac{1}{\sqrt{\frac{2}{4}}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

8. $\boxed{E}$ $f(x) = e^{x^3-2x^2-4x+5}$ $\quad f'(x) = e^{x^3-2x^2-4x+5}(3x^2-4x-4)$

$f'(x) = 0$ when $3x^2-4x-4=0$ $\quad (3x+2)(x-2)=0$ @ $x=2$, $x=-2/3$

$f'(x)$ defined everywhere

	(-1)	$\boxed{-2/3}$	(0)	$\boxed{2}$	(3)
f'	$+(-)(-)=+$		$+(+)(-)=-$		$+(+)(+)$
f	inc		dec		inc

min @ $x=2$

9. $\boxed{D}$ normal line: $\perp$ to graph $y = \frac{3x}{x^2-6}$ so need negative reciprical

$\frac{27}{2}$ 54

$y' = \frac{(x^2-6)(3) - 3x(2x)}{(x^2-6)^2}\Big|_{x=3} = \frac{(9-6)(3) - 3(3)(2)(3)}{(9-6)^2} = \frac{9-54}{9} = \frac{-45}{9} = \boxed{-5}$

normal line slope: $-\frac{1}{-5} = \frac{1}{5}$ $\quad$ so $y-3 = \frac{1}{5}(x-3)$

$5y - 15 = x - 3$

@ $x=3$ $\quad y = \frac{9}{9-6} = \frac{9}{3} = 3$ $\quad \boxed{5y - x = 12}$

10. $\boxed{A}$ $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$ is the limit defn of derivative of x^2

so $g(x) = 2x$. $\quad g(x) = 2$ when $x=1$

11. $\boxed{E}$ $6x + 2x\frac{dy}{dx} + y(2) + 2y\frac{dy}{dx} = 0$ $\quad$ ← plug in $\quad$ to find y: $3 + 2y + y^2 = 2$ $\quad (x=1)$

plug in $(1,-1)$ $\quad 6 + 2\frac{dy}{dx} + -2\frac{dy}{dx} = 0$ $\quad y^2 + 2y + 1 = 0$

$6 = 0$? not defined! $\quad (y+1)(y+1) = 0$ $\quad y = -1$

12. $\boxed{B}$ $f(x) = x^3 - 12x$ $\quad f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x+2)(x-2)$

$f''(x) = 6x$

	(-3)	$\boxed{-2}$	(0)	$\boxed{2}$	(3)
f'	$-(-)=+$		$+(-)=-$		$+(+)=+$
f	inc		dec		inc
		C true / B false		A true	

	-1	0	1
f''	$-$		$+$
f''	concave↓	E true	concave↑ D true

17 $\boxed{B}$ position: $s(t)=t^2+4t+4$
velocity: $v(t)=s'(t)=2t+4$
acceleration: $a(t)=v'(t)=s''(t)=2$ for all time

13. $\boxed{A}$ $f(x)=\dfrac{x^2-x-6}{x^2-5x+6}=\dfrac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-2)}\approx\dfrac{x+2}{x-2}$

$f(3)=\lim\limits_{x\to3}\dfrac{x+2}{x-2}=\dfrac{3+2}{3-2}=\boxed{5}$

14. $\boxed{E}$ $f(x)=[\cos(2x)]^3$ $\quad f'(x)=3(\cos(2x))^2\cdot(-\sin(2x))(2)$
$f'(x)=-6\sin(2x)\cdot\cos^2(2x)$

15. $\boxed{C}$ f' increasing $\Leftrightarrow$ $f''+$ $\Leftrightarrow$ f concave up
III

16. $\boxed{B}$ $f(x)=x^3-6x^2+9x-4$ $\quad f'(x)=3x^2-12x+9$
$f''(x)=6x-12$ $\quad f'(x)=3(x^2-4x+3)=3(x-3)(x-1)$
2nd deriv test! $f''(3)=18-12=6$ $\cup$ min @ $x=3$ $\quad f'(x)=0$ @ $x=3,1$
$f''(1)=6-12=-6$ $\cap$ max @ $x=1$ $\quad$ (could do sign chart w/ f' also!)

18. $\boxed{B}$ pts of inflection $\Rightarrow$ y'' changes sign

$y=\frac{1}{10}x^5+\frac{1}{2}x^4-\frac{3}{10}$ $\quad y'=\frac{5}{10}x^4+\frac{4}{2}x^3$ $\quad y''=\frac{20}{10}x^3+\frac{12}{2}x^2$

$y''=2x^3+6x^2=2x^2(x+3)=0$ @ $x=0,\ x=-3$

(-4) $\boxed{-3}$ (-1) $\boxed{0}$ (1)

f''	$+(-)=-$	$+(+)=+$	$+(+)=+$
f	concave ↓	concave ↑	concave ↑

f'' Δ's sign so f changes concavity @ $x=-3$

19. $\boxed{C}$
A. not true b/c f is continuous
B. not true b/c f is continuous (defn of continuity)
C. True! f could be cont. @ a cusp, but not diff!
D. not true b/c f is cont.

20. $\boxed{A}$ $x^3+2x^2y-4y=7$ $\quad 3x^2+2x^2\frac{dy}{dx}+y(4x)-4\frac{dy}{dx}=0$
plug in $(1,-3)$

when $x=1$ $\quad 1+2y-4y=7$
$-2y=6$
$\frac{-2y}{-2}=\frac{6}{-2}$
$y=-3$

$3+2\cdot\frac{dy}{dx}+\underbrace{(-3)(4)}_{-12}-4\frac{dy}{dx}=0$

$-2\frac{dy}{dx}=9$ $\quad \frac{dy}{dx}=-\frac{9}{2}$

Go to next page for #22 – #32

21. $\boxed{E}$ I. $f(x) < 0$ for all x. ($f(x)$ is negative)
II. $g(5) = 2$.

need $f(x)g'(x) = 0$

$h(x) = \frac{f(x)}{g(x)}$ $h'(x) = \frac{g(x) \cdot f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{f'(x)}{g(x)}$

D: can't be since $g(5) = 2$.

since $f(x) < 0$ $g'(x) = 0$
best answer: $g(x) = 2$

Calculator

33. $\boxed{E}$ exponential always grows fastest as $x \to \infty$

34 $\boxed{E}$ $v(x) = 3x^2 - 4x$ acc $= v'(x) = 6x - 4$

35. $\boxed{E}$ $f'(3.6) \approx \frac{f(3.7) - f(3.5)}{3.7 - 3.5} = \frac{4.89 - 4.25}{.2} = 3.2$

36. $\boxed{C}$ I. $\lim_{x \to 1} f(x)$ exists? No! $\lim_{x \to 1^+} f(x) = 6(1) - 5 = 1$ $\neq$!! $\lim_{x \to 1^-} f(x) = 3 - 4 = -1$

II. $f'(1)$ exists no! b/c I not true.

$\boxed{III}$. $\lim_{x \to 1^-} f'(x) = 6x \Rightarrow 6$ $\lim_{x \to 1^+} f'(x) = 6$ yes!

37. $\boxed{B}$ $x_1(t) = 2t^2 - 5t + 17$ $x_2(t) = \sin(2t)$
$v_1(t) = 4t - 5$ $v_2(t) = \cos(2t) \cdot 2 = 2\cos(2t)$
$a_1(t) = 4$ $a_2(t) = -2\sin(2t)(2) = -4\sin(2t)$

want $a_1(t) = \underbrace{4}_{y_1} = \underbrace{-4\sin(2t)}_{y_2} = a_2(t)$ graph on window $[0,5]$ t
$[-5,5]$ y

only intersect 1 time

38. $\boxed{D}$ critical pts: $f'(x) = 0$ $f'(x)$ UD $\leftarrow$ none of these based on $f'(x)$ eqn.

$f'(x) = \underbrace{e^x - 3x^2}_{\text{graph this + count x-intercepts: 3}} = 0$

22. $\boxed{B}$ $\lim_{x\to 1^+} f(x) = \lim_{x\to 1} -\frac{1}{2}x+1 = -\frac{1}{2}+1 = \boxed{\frac{1}{2}}$

23. $\boxed{E}$ $\lim_{x\to 3^+} \frac{x+3}{x-3}$ $\quad 3^+ \approx 3.01$ $\quad$ so $\lim_{x\to 3^+} \approx \frac{6.01}{3.01-3} = \frac{6.01}{.01} \to \infty$

as you plug in x-values closer + closer to $x=3$, the denominator gets to be a smaller fraction, making $\frac{x+3}{x-3}$ larger

24. $\boxed{a=-2}$ continuous: $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$ $\quad 5-a = 4+3 = 4+3$

$5-a=7$ $\boxed{a=-2}$

25. $\boxed{C}$ $f(x) = (x-1)^{3/2} + \frac{1}{2}e^{x-2}$ $\qquad f'(x) = \frac{3}{2}(x-1)^{1/2} + \frac{1}{2}e^{x-2}$

$f'(2) = \frac{3}{2}(1)^{1/2} + \frac{1}{2}e^0 = \frac{3}{2} + \frac{1}{2} = 2$

26. $\boxed{D}$ $f(x) = (x-1)^2 \sin x$ $\qquad f'(x) = (x-1)^2 \cdot \cos x + \sin x\,[2(x-1)']$

$f'(0) = (-1)^2 \cos(0) + \sin(0)[2(-1)] = 1 + 0 = 1$

27. $\boxed{A}$ $y = [\cos(3x)]^2$ $\qquad y' = 2[\cos(3x)](-\sin(3x)) \cdot 3 = -6\cos(3x)\sin(3x)$

28. $\boxed{E}$ $y = \cos(2x)$ $\quad y = \cos(2(\pi/4)) = \cos(\pi/2) = 0$ $\qquad y' = -\sin(2x)\cdot 2 \,\Big|_{x=\pi/4}$

$y - 0 = -2(x - \pi/4)$ $\qquad y' = -2\sin(\pi/2) = \boxed{-2}$

29. $\boxed{C}$ $y = 2^x$ $\quad y' = 2^x \ln 2$

30. $\boxed{A}$ $y = 3(4+x^2)^{-1}$ $\qquad y' = -3(4+x^2)^{-2}\cdot(2x) = \frac{-6x}{(4+x^2)^2}$

31. $\boxed{B}$ $\quad y' > 0$ (y increasing) $\quad y'' < 0$ (y concave ↓)

32. $\boxed{C}$ $\quad f'(x) = 3x^2 + 12 = 3(x^2+4) = 0$ never; VD never

$f'(x) > 0$ for all x so f increasing for all x

(39). $\boxed{B}$ $f'(x)=5x+1$

$f(x)=x^5+x$ $\qquad \frac{d}{dx} f^{-1}(2)=\dfrac{1}{f'(f^{-1}(2))}=\dfrac{1}{f'(1)}=\dfrac{1}{6}$

$f^{-1}(2)=?$

$f(?)=2$ $\qquad f(1)=2$ so $f^{-1}(2)=1$

(40). $\boxed{D}$ $f(x)=x^2-3\sqrt{x+2}$ use min calculator: 2nd Trace →

minimum $x=.47657$

(41) $\boxed{A}$ I. $f(x)$ decreasing on (a,c) yes! b/c f' is negative

II. $f(x)$ concave up on (a,c) no! f concave ↑ when f' inc.

III. $f(x)$ rel min @ $x=a$ no! f has rel max @ $x=a$

since f' changes from $+ \to -$

(42) $\boxed{D}$ $f'(2)=g'(h(2))\cdot h'(2)=g'(3)\cdot 4=5\cdot 4=20$

(43). $\boxed{B}$ parallel tangents means = slopes (derivatives)

$f'(x)=6e^{3x}$ $\quad g'(x)=15x^2$ $\quad$ graph $\underbrace{6e^{3x}}_{y_1}=\underbrace{15x^2}_{y_2}$

window: $[-1,0]\times[-10,10]$

calculate intersect (2nd Trace) @ $x=-.3655$

(44). $\boxed{B}$ $f'(x)=\dfrac{\sin^2 x}{x}-2/9$ $\quad$ critical pts where $f'(x)$ UD → none

not undefined on $(0,10)$

where $f'(x)=0$

graph $f'(x)$ on $[0,10]$ + count how many times $f'(x)$ crosses the x-axis. - 2 times

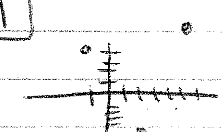
(45) $\boxed{A}$ $\quad m=-1$

$f'(x)=\underbrace{6x^5-4x^3}_{y_1}=\underbrace{-1}_{y_2}$ @ $x=-.93355$

$f(-.93355)=-.097589$

$y+.097589=-1(x+.93355)$ $\quad y=-x-.93355-.097589$

$\boxed{y=-x-1.0311}$



(46). $\boxed{D}$ $\quad f(-1)=4 \quad f(2)=-5 \quad f(6)=8$

I yes by IVT $\quad$ III yes! since 4 is between -5 and 8

II not necessarily, we don't know any other f values

(47) $f'(a)$ I: $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ True

$\boxed{C}$ II: $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ True

III $\lim_{x \to a} \frac{f(x+h)-f(x)}{h}$ False need den $\to 0$.

(48) $\boxed{A}$ $\lim_{h \to 0} \frac{\ln(e+h)-1}{h} = \lim_{h \to 0} \frac{\ln(e+h)-\ln(e)}{h}$ $f(x) = \ln x$
$a = e$

this is saying find $f'(e)$. $f'(x) = 1/x \,|_e = \boxed{1/e}$

(49) $\boxed{B}$ not diff @ v. tangents: $x=2$
@ discontinuities: $x=0$

(50) $\boxed{D}$ $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$ position

$v(t) = y'(t) = -\frac{1}{6}\sin(5t)\cdot 5 - \frac{1}{4}\cos(5t)\cdot 5$

$v(t) = -\frac{5}{6}\sin(5t) - \frac{5}{4}\cos(5t)$ velocity

Graph velocity + count how many times it touches x-axis between $t=0$ and $t=4$. <u>6 times!</u>