3.3 Graphing and Solving Systems of Linear Inequalities

GOAL 1 GRAPHING A SYSTEM OF INEQUALITIES

The following is a **system of linear inequalities** in two variables.

$$x + y \leq 6 \qquad \text{Inequality 1}$$
$$2x - y > 4 \qquad \text{Inequality 2}$$

A **solution** of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system. For example, $(3, -1)$ is a solution of the system above. The **graph** of a system of linear inequalities is the graph of all solutions of the system.
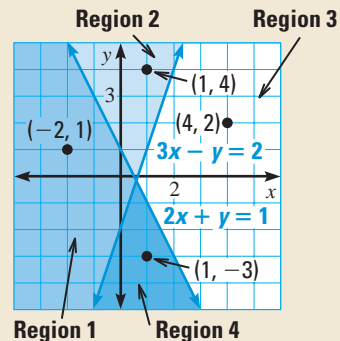
▶ **ACTIVITY**

Developing Concepts

Investigating Graphs of Systems of Inequalities

The coordinate plane shows the four regions determined by the lines $3x - y = 2$ and $2x + y = 1$. Use the labeled points to help you match each region with one of the systems of inequalities.

a. $3x - y \leq 2$
$2x + y \leq 1$

b. $3x - y \geq 2$
$2x + y \geq 1$

c. $3x - y \geq 2$
$2x + y \leq 1$

d. $3x - y \leq 2$
$2x + y \geq 1$

As you saw in the activity, a system of linear inequalities defines a region in a plane. Here is a method for graphing the region.

GRAPHING A SYSTEM OF LINEAR INEQUALITIES

To graph a system of linear inequalities, do the following for each inequality in the system:

- Graph the line that corresponds to the inequality. Use a dashed line for an inequality with $<$ or $>$ and a solid line for an inequality with $\leq$ or $\geq$.

- Lightly shade the half-plane that is the graph of the inequality. Colored pencils may help you distinguish the different half-planes.

The graph of the system is the region common to all of the half-planes. If you used colored pencils, it is the region that has been shaded with *every* color.

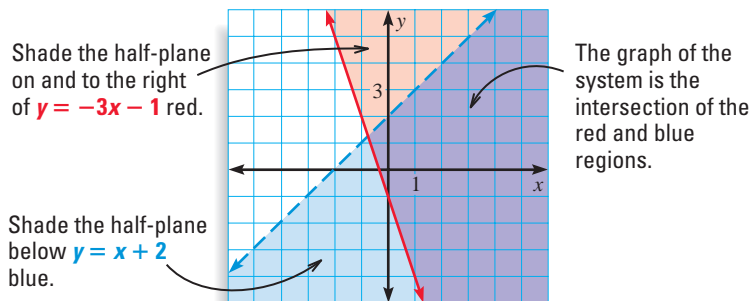**EXAMPLE 1**   *Graphing a System of Two Inequalities*

Graph the system.

$$y \geq -3x - 1 \qquad \textbf{Inequality 1}$$
$$y < x + 2 \qquad \textbf{Inequality 2}$$

SOLUTION

Begin by graphing each linear inequality. Use a different color for each half-plane. For instance, you can use red for Inequality 1 and blue for Inequality 2. The graph of the system is the region that is shaded purple.

Shade the half-plane on and to the right of $y = -3x - 1$ red.

The graph of the system is the intersection of the red and blue regions.

Shade the half-plane below $y = x + 2$ blue.

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You can also graph a system of three or more linear inequalities.

EXAMPLE 2   *Graphing a System of Three Inequalities*

Graph the system.

$$x \geq 0 \qquad \textbf{Inequality 1}$$
$$y \geq 0 \qquad \textbf{Inequality 2}$$
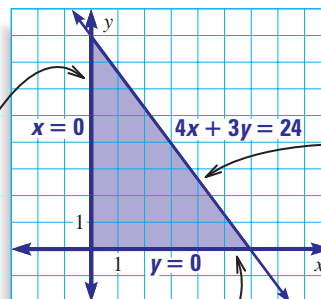$$4x + 3y \leq 24 \qquad \textbf{Inequality 3}$$

SOLUTION

Inequality 1 and Inequality 2 restrict the solutions to the first quadrant. Inequality 3 is the half-plane that lies on and below the line $4x + 3y = 24$. The graph of the system of inequalities is the triangular region shown below.

STUDENT HELP

▶ **Study Tip**
From this point on, only the solution region will be shaded on graphs of systems of linear inequalities.

The inequality $x \geq 0$ implies that the region is on and to the right of the *y*-axis.

The inequality $4x + 3y \leq 24$ implies that the region is on and below the line $4x + 3y = 24$.

The inequality $y \geq 0$ implies that the region is on and above the *x*-axis.

REAL LIFE **PERSONAL TRAINER**

A personal trainer can help you assess your fitness level and set exercise goals. As described in Example 3, one way to do this is by monitoring your heart rate.

CAREER LINK
www.mcdougallittell.com

GOAL 2 USING SYSTEMS OF INEQUALITIES IN REAL LIFE

You can use a system of linear inequalities to describe a real-life situation, as shown in the following example.

EXAMPLE 3 *Writing and Using a System of Inequalities*

HEART RATE A person's theoretical maximum heart rate is $220 - x$ where x is the person's age in years ($20 \leq x \leq 65$). When a person exercises, it is recommended that the person strive for a heart rate that is at least 70% of the maximum and at most 85% of the maximum.

a. You are making a poster for health class. Write and graph a system of linear inequalities that describes the information given above.

b. A 40-year-old person has a heart rate of 150 (heartbeats per minute) when exercising. Is the person's heart rate in the target zone?

SOLUTION

a. Let y represent the person's heart rate. From the given information, you can write the following four inequalities.

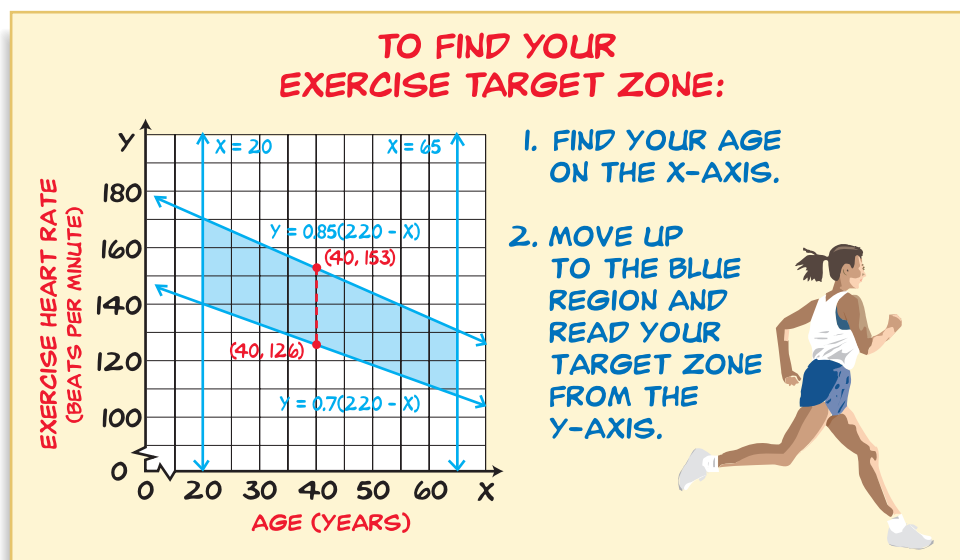|	
$x \geq 20$	**Person's age must be at least 20.**
$x \leq 65$	**Person's age can be at most 65.**
$y \geq 0.7(220 - x)$	**Target rate is at least 70% of maximum rate.**
$y \leq 0.85(220 - x)$	**Target rate is at most 85% of maximum rate.**

The graph of the system is shown below.

TO FIND YOUR
EXERCISE TARGET ZONE:

1. FIND YOUR AGE ON THE X-AXIS.

2. MOVE UP TO THE BLUE REGION AND READ YOUR TARGET ZONE FROM THE Y-AXIS.

b. From the graph you can see that the target zone for a 40-year-old person is between 126 and 153, inclusive. That is,

$$126 \leq y \leq 153.$$

▶ A 40-year-old person who has a heart rate of 150 is within the target zone.

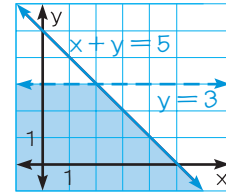# GUIDED PRACTICE

Vocabulary Check ✓

1. What must be true in order for an ordered pair to be a solution of a system of linear inequalities?

Concept Check ✓

2. Look back at Example 1 on page 157. Explain why the ordered pair $(-1, -5)$ is *not* a solution of the system.

3. **ERROR ANALYSIS** Explain what is wrong with the graph of the following system of inequalities.

$$y \leq 3$$
$$x + y \geq 5$$

Skill Check ✓

Tell whether the ordered pair is a solution of the following system.

$$x \geq -1$$
$$y > 2x + 2$$

4. $(-1, 2)$ **5.** $(0, 0)$ **6.** $(1, 4)$ **7.** $(2, 7)$

Graph the system of linear inequalities.

8. $x \geq -1$
   $y > 2x + 2$

9. $x + y \leq 3$
   $y > 1$

10. $x > 0$
    $y \leq x - 5$

11. 🌐 **FLIGHT ATTENDANTS** To be a flight attendant, you must be at least 18 years old and at most 55 years old, and you must be between 60 and 74 inches tall, inclusive. Let x represent a person's age (in years) and let y represent a person's height (in inches). Write and graph a system of linear inequalities showing the possible ages and heights for flight attendants.
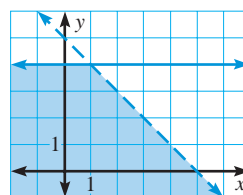
PRACTICE AND APPLICATIONS

CHECKING A SOLUTION Tell whether the ordered pair is a solution of the system.

12. $(25, -5)$
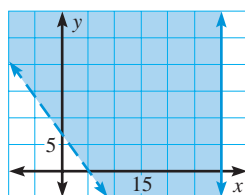
13. $(2, 3)$
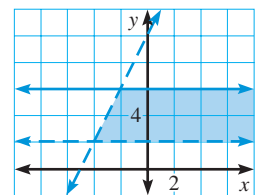

14. $(2, 6)$

FINDING A SOLUTION Give an ordered pair that is a solution of the system.

15. $x - y \geq 3$
    $y < 15$

16. $x + y < 6$
    $x \geq -2$

17. $4x > y$
    $x \leq 12$

18. $x \geq -7$
    $y < 10$
    $x < y$

19. $y > -5$
    $x > 3$
    $2x + y < 13$

20. $y \geq -x$
    $y \geq 0$
    $x < 0$

MATCHING SYSTEMS AND GRAPHS Match the system of linear inequalities with its graph.

21. $y \le 4$
$x > -2$

22. $y > -4$
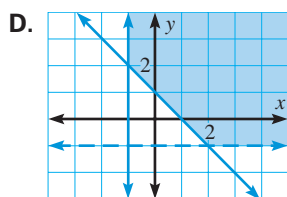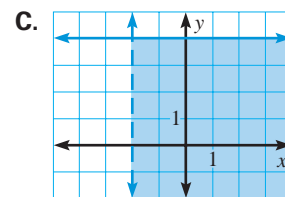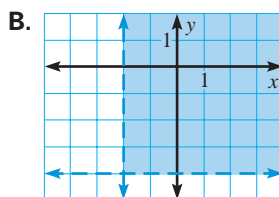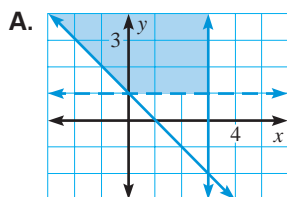$x > -2$

23. $y > x$
$x > -3$
$y \ge 0$

24. $y > x$
$y > -3$
$x \le 0$

25. $x \le 3$
$y > 1$
$y \ge -x + 1$

26. $y > -1$
$x \ge -1$
$y \ge -x + 1$

A.

B.

C.

D.

E.

F.
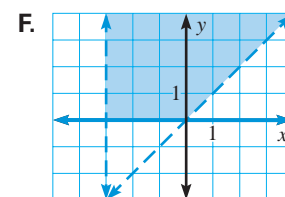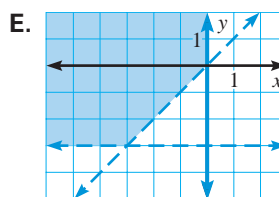
SYSTEMS OF TWO INEQUALITIES Graph the system of linear inequalities.

27. $x < 5$
$x > -4$

28. $y > -2$
$y \le 1$

29. $x \ge 0$
$x + y < 11$

30. $x + y \ge -2$
$-5x + y < -3$

31. $y \ge -4$
$y < -2x + 10$

32. $y > 2x - 7$
$4x + 4y < -12$

33. $y < x + 4$
$y \ge -2x + 1$

34. $x + y > -8$
$x + y \le 6$

35. $y > -3x$
$x \le 5y$

36. $x - y > 7$
$2x + y < 8$

37. $7x + y > 0$
$3x - 2y \le 5$

38. $-x < y$
$x + 3y > 8$

SYSTEMS OF THREE OR MORE INEQUALITIES Graph the system of linear inequalities.

39. $y < 4$
$x > -3$
$y > x$

40. $y \ge 1$
$x \le 6$
$y < 2x - 5$

41. $2x - 3y > -6$
$5x - 3y < 3$
$x + 3y > -3$

42. $x - 4y > 0$
$x + y \le 1$
$x + 3y > -1$

43. $2x + 1 \ge y$
$x < 5$
$y < x + 2$

44. $5x - 3y \le 4$
$x + y < 8$
$y > 3$

45. $x \ge y - 2$
$x + y > 1$
$x < 10$

46. $y \ge 0$
$x - 4y < 2$
$y < x$

47. $x - y \ge 0$
$y < 2x$
$5x + 6y \ge 1$

48. $y \ge 0$
$x \le 9$
$x + y < 15$
$y < x$

49. $x + y \le 4$
$x + y \ge -1$
$x - y \ge -2$
$x - y \le 2$

50. $y < 5$
$y > -6$
$2x + y \ge -1$
$y \le x + 3$

51. 🌐 **POOL CHEMICALS** You are a lifeguard at a community pool, and you are in charge of maintaining the proper pH (amount of acidity) and chlorine levels. The water test-kit says that the pH level should be between 7.4 and 7.6 pH units and the chlorine level should be between 1.0 and 1.5 PPM (parts per million). Let p be the pH level and let c be the chlorine level (in PPM). Write and graph a system of inequalities for the pH and chlorine levels the water should have.

HEALTH **In Exercises 52–54, use the following information.**
For a healthy person who is 4 feet 10 inches tall, the recommended lower weight limit is about 91 pounds and increases by about 3.7 pounds for each additional inch of height. The recommended upper weight limit is about 119 pounds and increases by about 4.9 pounds for each additional inch of height.
▶ Source: Dietary Guidelines Advisory Committee

52. Let x be the number of inches by which a person's height exceeds 4 feet 10 inches and let y be the person's weight in pounds. Write a system of inequalities describing the possible values of x and y for a healthy person.

53. Use a graphing calculator to graph the system of inequalities from Exercise 52.

54. What is the recommended weight range for someone 6 feet tall?

🌐 **SHOE SALE** **In Exercises 55 and 56, use the shoe store ad shown below.**

55. Let x be the regular footwear price and y be the discount price. Write a system of inequalities for the regular footwear prices and possible sale prices.

56. Graph the system you wrote in Exercise 55. Use your graph to estimate the range of possible sale prices for shoes that are regularly priced at $65.

HUGE
ONE-DAY SALE!
Save
10%–25%
on all athletic footwear.
(Regular price: $20–$80)

57. 🌐 **WEIGHTLIFTING RECORDS** The men's world weightlifting records for the 105-kg-and-over weight category are shown in the table. The combined lift is the sum of the snatch lift and the clean and jerk lift. Let s be the weight lifted in the snatch and let j be the weight lifted in the clean and jerk. Write and graph a system of inequalities to describe the weights you could lift to break the records for both the snatch and combined lifts, but *not* the clean and jerk lift.

Men's +105 kg World Weightlifting Records		
Snatch	Clean & Jerk	Combined
205.5 kg	262.5 kg	465.0 kg

INTERNET
DATA UPDATE of International Weightlifting Federation data at www.mcdougallittell.com

58. **BIOLOGY** **CONNECTION** Each day, an average adult moose can process about 32 kilograms of terrestrial vegetation (twigs and leaves) and aquatic vegetation. From this food, it needs to obtain about 1.9 grams of sodium and 11,000 Calories of energy. Aquatic vegetation has about 0.15 gram of sodium per kilogram and about 193 Calories of energy per kilogram, while terrestrial vegetation has minimal sodium and about four times more energy than aquatic vegetation. Write and graph a system of inequalities describing the amounts t and a of terrestrial and aquatic vegetation, respectively, for the daily diet of an average adult moose.
▶ Source: *Biology by Numbers*

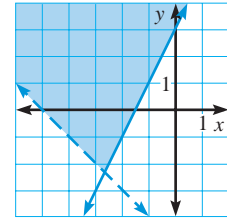**59. CRITICAL THINKING** Write a system of three linear inequalities that has no solution. Graph the system to show that it has no solution.

Test Preparation

60. MULTIPLE CHOICE Which system of inequalities is graphed at the right?

(A) $x + y > -5$
$\quad\quad -2x + y \geq 3$

(B) $x + y > -5$
$\quad\quad -2x + y < 3$

(C) $x + y > -5$
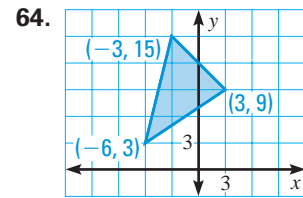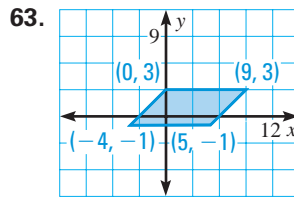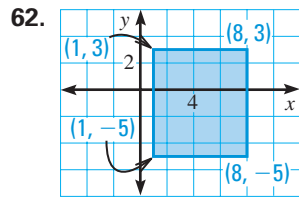$\quad\quad -2x + y \leq 3$

(D) $x + y > -5$
$\quad\quad -2x + y > 3$

61. MULTIPLE CHOICE Which ordered pair is *not* a solution of the following system of inequalities?

$$3x + 2y \geq -2$$
$$x - y < 3$$

(A) $(0, 0)$ $\quad\quad$ (B) $(-1, 2)$ $\quad\quad$ (C) $(4, 1)$ $\quad\quad$ (D) $(2, 2)$

★ Challenge

WRITING A SYSTEM Write a system of linear inequalities for the region.

62.

63.

64.

65. VISUAL THINKING Write a system of linear inequalities whose graph is a pentagon and its interior.
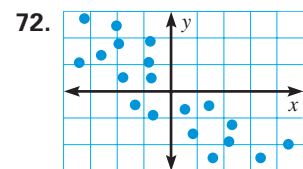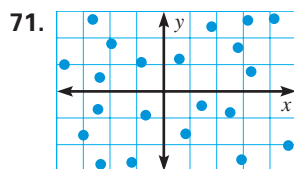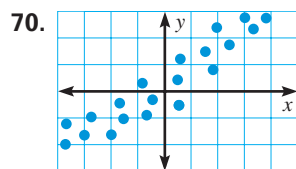
MIXED REVIEW

EVALUATING EXPRESSIONS Evaluate the expression for the given values of *x* and *y*. **(Review 1.2 for 3.4)**

66. $2x + 7y$ when $x = 5$ and $y = -3$ $\quad\quad$ **67.** $-4x - 3y$ when $x = -6$ and $y = -1$

68. $10x - 3y$ when $x = -4$ and $y = 2$ $\quad\quad$ **69.** $-y + 8x$ when $y = -3$ and $x = -2$

DETERMINING CORRELATION Tell whether *x* and *y* have a *positive correlation*, a *negative correlation*, or *relatively no correlation*. **(Review 2.5)**

70.

71.

72.

CHOOSING A METHOD Solve the system using any algebraic method. **(Review 3.2)**

73. $13x + 5y = 2$
$\quad\quad x - 4y = 10$

74. $-2x + 7y = 10$
$\quad\quad x - 3y = -3$

75. $5x + 6y = -12$
$\quad\quad 10x + 12y = 24$

76. $-7x + 5y = 0$
$\quad\quad 14x - 8y = 2$

77. $-4x - 10y = 12$
$\quad\quad x + 5y = 2$

78. $6x - 8y = -18$
$\quad\quad -3x + 4y = 9$