The function f is twice differentiable. Select values of f and f' are given in the table below. f has exactly two critical numbers on the interval $(1, 12)$.

x	1	2	5	7	10	12
$f(x)$	2	4	2	1	7	9
$f'(x)$	1	0	-2	0	$3/2$	1

a) Use the table to approximate $f''(6)$.

$$f''(6) \approx \frac{f'(7) - f'(5)}{7 - 5} = \frac{0 - (-2)}{7 - 5} = \frac{2}{2} = 1$$

b) Use a trapezoidal sum of four subintervals to approximate $\int_1^{10} f(x)\,dx$.

Left Riemann: $(2-1)(2) + (5-2)(4) + (7-5)(2) + (10-7)(1) = 21$

Right Riemann: $(2-1)(4) + (5-2)(2) + (7-5)(1) + (10-7)(7) = 33$

Average of Left and Right Riemann: $\frac{21 + 33}{2} = 27$

-OR-

Trapezoidal Sum: $\frac{1}{2}(2-1)(2+4) + \frac{1}{2}(5-2)(4+2) + \frac{1}{2}(7-5)(2+1) + \frac{1}{2}(10-7)(1+7) = 27$

c) Find $\int_1^{12} (f'(x) + 2)\,dx$.

$$\int_1^{12} (f'(x) + 2)\,dx = \int_1^{12} f'(x)\,dx + \int_1^{12} (2)\,dx = f(12) - f(1) + 11(2) = 9 - 2 + 22 = 29$$

d) Verify $\lim_{x \to 12} \frac{\int_1^x (f'(t) + 2)\,dt - 29}{6x - 72} = \frac{1}{2}$

$$\lim_{x \to 12} \left(\int_1^x (f'(t) + 2)\,dt - 29 \right) = 29 - 29 = 0 = \lim_{x \to 12} (6x - 72)$$

By L'Hospital's Rule: $\lim_{x \to 12} \frac{\int_1^x (f'(t) + 2)\,dt - 29}{6x - 72} = \lim_{x \to 12} \frac{f'(x) + 2}{6} = \frac{f'(12) + 2}{6} = \frac{1 + 2}{6} = \frac{3}{6} = \frac{1}{2}$

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e) Write an equation of the tangent line to f at $x = 10$. Use this line to approximate $f(11)$.

$$m = f'(10) = \frac{3}{2}; \quad f(10) = 7$$

$$y = 7 + \frac{3}{2}(x - 10)$$

$$f(11) \approx 7 + \frac{3}{2}(11 - 10) = 8\frac{1}{2}$$

f) What is the least number of times $f(x) = 3$ on $(1, 12)$? Explain your reasoning.

3 times.
Since f is twice differentiable, it is continuous.

For $1 < x < 2$, $f(1) = 2 < 3 < 4 = f(2)$.
For $2 < x < 5$, $f(2) = 4 > 3 > 2 = f(5)$.
For $7 < x < 10$, $f(7) = 1 < 3 < 7 = f(10)$.

So, there must be at least three values of x such that $f(x) = 3$.
(This is use of the Intermediate Value Theorem)

g) Let $k(x) = f(2x)$. Find $k'(5)$.

$$k'(x) = f'(2x) \cdot 2$$

$$k'(5) = f'\big(2(5)\big) \cdot 2 = f'(10) \cdot 2 = \frac{3}{2} \cdot 2 = 3$$

h) Let $m(x) = f^{-1}(x)$, the inverse of f on the interval $(7, 12)$. Find $m'(7)$.

$$m(7) = f^{-1}(7) = 10; \qquad m'(7) = \frac{1}{f'\big(m(7)\big)} = \frac{1}{f'(10)} = \frac{1}{3/2} = \frac{2}{3}$$

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i) There is a gecko scurrying along a straight path such that the velocity of the gecko, $v = f'$, where velocity is measured in centimeters per second. Find the average acceleration of the gecko on the interval [5, 12]. Indicate units of measure.

Average acceleration is average rate of change of velocity.

$$\text{Average acceleration} = \frac{v(12) - v(5)}{12 - 5} = \frac{f'(12) - f'(5)}{12 - 5} = \frac{1 - (-2)}{12 - 5} = \frac{3}{7}\ \text{cm/sec}^2$$

j) Explain why there must be a time in the interval [5, 12] where the acceleration of the gecko will be equal to the value found in (i).

$a(t) = v'(t) = f''(x)$.

Since f is twice differentiable, $v = f'$ is differentiable on (5, 12) and continuous on [5, 12].

By Mean Value Theorem there must be a value $x = c$ for $5 < c < 12$ such that

$$a(c) = v'(c) = \frac{v(12) - v(5)}{12 - 5} = \frac{3}{7}.$$