10.6 The Ratio Test

There is one last major test for convergence that is useful (as we shall see).

CONVERGENCE TEST:

Let $\sum a_n$ be a series with positive or negative terms, and with $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L$

Then, the series *converges* is L<1

The series *diverges* if L>1

The test is inconclusive if L=1

This is called the **Ratio Test**.

This test is one of the most useful for analyzing various facets of series. It also gives us a way to analyze the convergence of series that would otherwise be challenging using the other tests!

Example: Determine if the following series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$$

$$\sum_{n=1}^{\infty} \frac{n!}{5^n}$$

Practice: Determine if the following series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n^3}{(2n-1)!}$$

$$\sum_{n=1}^{\infty} \frac{3^n}{(-2)^{n+1}n}$$

Example: Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$$

Example: For what values of x does the series below converge?

$$\sum_{n=0}^{\infty} \frac{nx^n}{10^n}$$

The width of the interval over which a series converges is called___________________________.

Practice: For what values of p does the series below converge?

$$\sum_{n=0}^{\infty} \frac{(p-1)^{2n}}{4^n}$$

Practice: Use the Ratio Test to determine if the series converges or diverges.

1. $\sum_{k=1}^{\infty} \frac{2^k}{k!}$

2. $\sum_{k=1}^{\infty} ke^{-k}$

Homework

Use the Ratio Test or the Alternating Series Test (AST) to determine if the series converges. If a test is inconclusive, use a different test to check convergence.

(i) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

(j) Ratio Test: $\sum_{n=1}^{\infty} \frac{2}{n^2}$

(k) AST: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

(l) AST: $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{2n}$

Plus: Watch the video and take notes on Taylor Polynomial. It might be the most beautiful thing I have ever seen…