Test V1

Name:

Name Answer Key Date ________________

Answer each question as fully as possible. Every part of every question is 4 points.

No Calculator

1. Consider the following system of equations with the solution $x = 2, y = -5$.

Equation A1: $y = x - 7$

Equation A2: $y = -2x - 1$

a. Solve this system of equations using the elimination or substitution method.

$x-7=-2x-1$
$3x-7=-1$
$3x=6$
$x=2$

$y=x-7$
$y=2-7$
$y=-5$
$\boxed{(2,-5)}$

b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x - 7$

Equation C2: $3y = -3x - 9$

What multiple of A2 was added to A1?

A2: $(y=-2x-1)2$
$2y=-4x-2$
$+y=x-7$
$3y=-3x-9$

we add $2A_2$ to A_1 to get $3y=-3x-9$

c. What is the solution to the system given in part (b)?

It will be $(2,-5)$. we haven't changed the solution by multiplying by a factor.

Test V1

Name:

Solve the following system of equations by elimination. Graph the equations to verify your solution. Mark on the graph that the solution from your graphical work matches your solution from your algebraic work.

2. $y = x - 5$
$-\ y = -2x + 4$

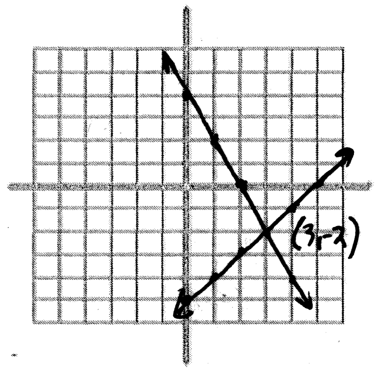
$0 = 3x - 9$

$+9 = 3x$

$+3 = x$

$y = 3 - 5$

$y = -2$ $(3, -2)$



3) Write the augmented matrix that corresponds to the following system of equations:

$$\begin{cases} x + y + z = 9 \\ 2x + 3y - z = 5 \\ x - 2y - z = -7 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 5 \\ 1 & -2 & -1 & -7 \end{bmatrix}$$

4) Use row operations to transform the matrix from Problem #3 into reduced row-echelon form to find the solutions of the system of equations.

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 5 \\ 1 & -2 & -1 & -7 \end{bmatrix} \xrightarrow{-R_1+R_3} \begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 5 \\ 0 & -3 & -2 & -16 \end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -13 \\ 0 & -3 & -2 & -16 \end{bmatrix} \xrightarrow{3R_2+R_3} \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -13 \\ 0 & 0 & -11 & -55 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{11}R_3} \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -13 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow{3R_3+R_2} \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow{-R_3+R_1} \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow{-R_2+R_1} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\boxed{(2, 2, 5)}$$

Test V1

Name:

5. If matrix $A=\begin{bmatrix} 3 & 4 \\ 6 & -2 \\ 1 & 0 \end{bmatrix}$ and matrix $B=\begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix}$, find $2A-5B$.

$$\begin{bmatrix} 6 & 8 \\ 12 & -4 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} -15 & 5 \\ 10 & -20 \\ -5 & 25 \end{bmatrix} = \begin{bmatrix} 21 & 3 \\ 2 & 16 \\ 7 & -25 \end{bmatrix}$$

6. If matrix $P=\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ and matrix $Q=\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$, find PQ.

$$\underset{2\times2}{\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}} \cdot \underset{2\times2}{\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}} = \underset{2\times2}{\begin{bmatrix} 9 & -8 \\ 5 & -4 \end{bmatrix}}$$

$3(-1)+4(3)=9$ $\quad$ $1(-1)+2(3)=5$

$3(0)+4(-2)=-8$ $\quad$ $1(0)+2(-2)=-4$

7. If matrix $A=\begin{bmatrix} 5 & 2 \\ -1 & 3 \end{bmatrix}$, find $|A|$. determinat

$5(3)-(-1)(2)$

$15+2$

17

Test V1

Name:

Calculator OK

8. A child's piggy bank contains only nickels, dimes, and quarters. There are 75 coins in the bank, and the value of the coins is $7.25. If there are five times as many nickels as dimes, find the number of coins of each type in the bank. You may solve this problem by getting the augmented matrix in reduced row-echelon form, by using the inverse of a matrix, ~~or by using~~

n: # of nickels d: # of dimes q: # of quarters

Total coins: $n + d + q = 75$

Total value: $5n + 10d + 25q = 725$

D/n: $-n + 5d = 0$

$5d = n$

$-n + 5d = 0$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 5 & 10 & 25 \\ -1 & 5 & 0 \end{bmatrix}} \underset{X}{\begin{bmatrix} n \\ d \\ q \end{bmatrix}} = \underset{B}{\begin{bmatrix} 75 \\ 725 \\ 0 \end{bmatrix}}$$

$$X = A^{-1}B = \begin{bmatrix} 50 \\ 10 \\ 15 \end{bmatrix}$$

$n = 50$

$d = 10$

$q = 15$

9. Use linear programming to find the solution requested. It may help to graph the situation.

The area of a parking lot is 600 square meters. A car requires 6 square meters. A bus requires 30 square meters. The attendant can handle only 60 vehicles. If a car is charged $2.50 and a bus $7.50, how many of each should be accepted to maximize income?

Variables: x = # of cars

y = # of buses

OF: $2.5(x) + 7.5(y) = I$

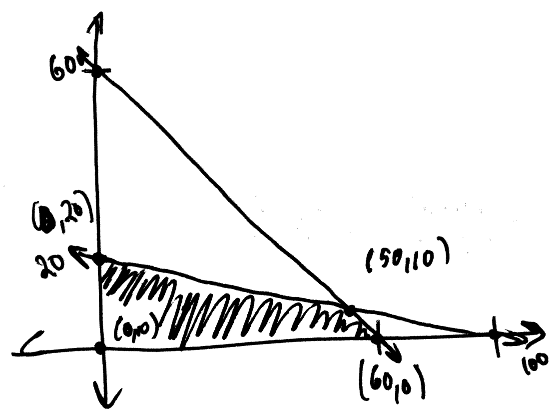
Constraints:

$6x + 30y \leq 600$

$x + y \leq 60$

$x \geq 0$

$y \geq 0$



(x,y)	(0,0)	(0,20)	(60,0)	(50,10)
I	0	150	150	200

$\boxed{\text{50 cars, 10 buses}}$

Test V1

Name:

10. Write a system of two equations in two variables where one equation is quadratic and the other is linear such that the system has two solutions. Explain, using graphs, algebra, and/or words, why the system has two solutions.

$y = x^2$ and $y = x$

$x = x^2$
$0 = x^2 - x$
$0 = x(x-1)$
$x = 0 \quad x = 1$ two solutions!

11. Use a matrix equation and inverse matrices to solve the following system of equations. You must write out the matrices you use and what you type in the calculator to find the solution.

$$s + t - u = 5$$
$$2s - 5t + 3u = 10$$
$$-s + 6t - 7u = 2$$

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 \\ 2 & -5 & 3 \\ -1 & 6 & -7 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} s \\ t \\ u \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 5 \\ 10 \\ 2 \end{bmatrix}}_{B}$$

$$X = A^{-1}B = \begin{bmatrix} 13/3 \\ -5/3 \\ -7/3 \end{bmatrix}$$

12.

SPORTS Two softball teams submit equipment lists for the season.

Women's team	Men's team
12 bats	15 bats
45 balls	38 balls
15 uniforms	17 uniforms

Each bat costs $21, each ball costs $4, and each uniform costs $30. Use matrix multiplication to find the total cost of equipment for each team.

$$\begin{array}{ccc} \text{bat} & \text{ball} & \text{uni} \end{array} \qquad \begin{array}{cc} W & M \end{array} \qquad \begin{array}{cc} \text{Wcost} & \text{Mcost} \end{array}$$

$$\begin{bmatrix} 21 & 4 & 30 \end{bmatrix} \times \begin{array}{c} \text{bat} \\ \text{ball} \\ \text{uni} \end{array} \begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix} = \begin{bmatrix} \$882 & \$977 \end{bmatrix}$$

Test V1

Name:

Excellence Points.

1. If matrix $B=\begin{bmatrix}5 & 2 & -1\\ 0 & 3 & 7\\ 6 & -8 & -4\end{bmatrix}$, find B^2.

$$B^2=\begin{bmatrix}19 & 24 & 13\\ 42 & -47 & -7\\ 6 & 20 & -46\end{bmatrix}$$

2. Solve for x and y.

$$\begin{bmatrix}-2 & 1 & 2\\ 3 & 2 & 4\\ 0 & -2 & 4\end{bmatrix}\begin{bmatrix}1\\ x\\ 3\end{bmatrix}=\begin{bmatrix}6\\ 19\\ y\end{bmatrix}$$

$-2(1)+1(x)+2(3)=6$

$-2+x+6=6$

$-2+x=0$

$x=2$

$0(1)+(-2)(2)+4(3)=y$

$-4+12=y$

$8=y$