Name: Answers Date:__________

Unit 2: Rates of Change, Derivatives and Differentiability Practice

1. Calculate the average rate of change of $f(x) = x^2 - 4x + 2$ over the interval [0, 4].

$$\frac{f(4)-f(0)}{4-0} = \frac{2-2}{4} = \boxed{0}$$

2. What are our two limit definitions of f '(a)? Write them out.

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a} \quad \text{OR} \quad \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

3. Evaluate $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$ when $f(x) = x^2 + 1$.

This is a derivative:

$f(x)=x^2+1$

$f'(x)=2x$ at $x=2$

$f'(2)=\boxed{4}$

OR

f(x) f(2)

$$\lim_{x \to 2} \frac{(x^2+1)-5}{x-2} = \lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} x+2 = \boxed{4}$$

4. Given $f(x) = -4x^2 + 4, \textit{determine } f'(-2)$

$f'(x)=-8x$

$f'(-2)=\boxed{16}$

Name:____Date:__________

5. $f(x) = \begin{cases} x^2 + 4x + 5, x < 1 \\ x^3 + bx + c, x \geq 1 \end{cases}$

If f(x) is continuous and differentiable at x=1, then what are the values of b and c?

Continuity:

$x^2+4x+5 = x^3+bx+c$ @ $x=1$

$1+4+5 = 1+b+c$

$4+5 = b+c$

$9 = b+c$

$9 = 3+c$

$\boxed{c=6}$

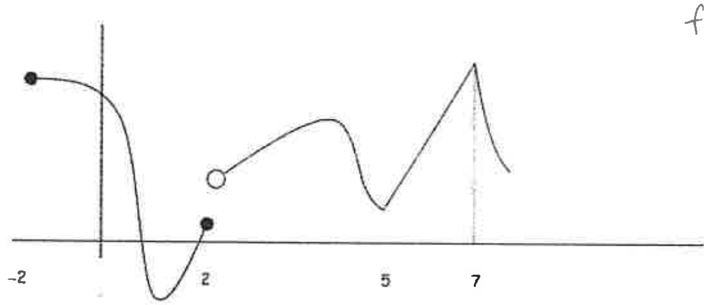
Differentiability:

$2x+4 = 3x^2+b$ @ $x=1$

$2+4 = 3+b$

$\boxed{3=b}$

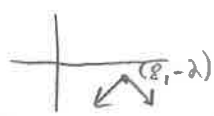
6. The graph of f is given below. State the numbers at which f is not differentiable and why. Your reason should be based on the definition of differentiability at a number.



f(x) is Not Differentiable at:

- $x=2$ b/c f(x) has a discontinuity at $x=2$
- $x=5$ b/c $\lim_{x\to5^-}\frac{f(x)-f(5)}{x-5} \neq \lim_{x\to5^+}\frac{f(x)-f(5)}{x-5}$ Cusp
- $x=7$ b/c $\lim_{x\to7^-}\frac{f(x)-f(7)}{x-7} \neq \lim_{x\to7^+}\frac{f(x)-f(7)}{x-7}$ Cusp

Absolute value graph



7. Let f(x)= -|x-8|-2. Which of the following statements about f are false?

I. f(x) is discontinuous at x= 8 Abs Val graphs are continuous

II. f(x) has a corner at x= 3 Corner at $x=3$

III. f(x) is differentiable at x= 8 No b/c corner at $x=8$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III only

Name:____Date:__________

8. Determine the value of f '(0) if $f(x) = \sin(x)\, e^x$.

$f'(x) = \sin(x) \cdot e^x + \cos(x) e^x$

$f'(0) = \sin(0) e^0 + \cos(0) e^0 = 0 + 1 = \boxed{1}$

9. If $f(x) = \cos(x)$, then the third derivative of f(x) at $x = \frac{\pi}{3}$ is

$f'(x) = -\sin(x)$

$f''(x) = -\cos(x)$

$f'''(x) = \sin(x) \longrightarrow f'''(\frac{\pi}{3}) = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

Quotient Rule

10. If $y = \frac{4^x}{\tan(x)}$, find the first derivative y' at $x = \frac{\pi}{4}$

$y' = \frac{\tan(x) \cdot 4^x \ln(4) - 4^x \sec^2(x)}{\tan^2(x)}$

at $x = \frac{\pi}{4}$ $\quad \frac{\tan(\frac{\pi}{4}) \cdot 4^{\pi/4} \ln(4) - 4^{\pi/4} \sec^2(\pi/4)}{\tan^2(\pi/4)} = \frac{1 \cdot 4^{\pi/4} \ln(4) - 4^{\pi/4} \cdot 2}{1^2} = 4^{\pi/4} \ln(4) - 4^{\pi/4} \cdot 2$

Quotient Rule

11. Find the derivative of $f(x) = \frac{x+4}{\ln(x)}$

$f'(x) = \frac{\ln(x)(1) - (x+4)\frac{1}{x}}{\ln(x)^2} = \frac{\ln(x) - (1 + \frac{4}{x})}{\ln(x)^2}$

Name:______________________________ Date:__________

12. $\frac{d}{dx}\cancel{\sec(x)}\cancel{\cos(x)}\cot(x) = \frac{d}{dx}\cot(x) = -\csc^2(x)$

$\frac{f(a+h)-f(a)}{h}$ $\Rightarrow$ $a = e$

$f(x) = \ln(x)$

13. $\lim_{h\to 0}\frac{\ln(e+h)-1}{h}$ is

$f'(e)$ is what this limit is calculating

(A) f ' (e), where f(x)=lnx

(B) f ' (e), where $f(x) = \frac{\ln x}{x}$

(C) f ' (1), where f(x)=lnx

(D) f ' (1), where f(x)=ln (x+e)

(E) f ' (0), where f(x)=lnx

14. Determine the tangent and normal line to the function $y = e^x + 2x$ at x=0.

Point: $x=0$ $\quad y = e^0 + 2(0) = 1$ $\quad (0,1)$

Slope: $\frac{dy}{dx} = e^x + 2$ at $x=0$ $\quad \frac{dy}{dx} = e^0 + 2 = 3$

T: $y - 1 = 3(x-0)$

N: $y - 1 = -\frac{1}{3}(x-0)$

15. If $y = \frac{x^2+x}{x^{-2}}$, then $\frac{dy}{dx} =$

Rewrite!

$y = \frac{x^2}{x^{-2}} + \frac{x}{x^{-2}} = x^4 + x^3$

$\frac{dy}{dx} = 4x^3 + 3x^2$

OR Quotient rule

$\frac{dy}{dx} = \frac{x^{-2}(2x+1) - (x^2+x)(-2x^{-3})}{(x^{-2})^2}$

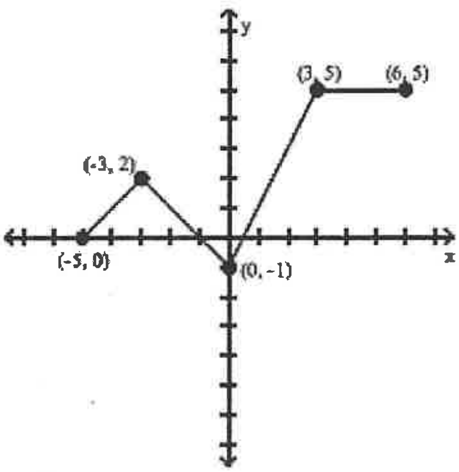
$= \frac{(2x^{-1}+x^{-2}) - (-2x^{-1} - 2x^{-2})}{x^{-4}}$

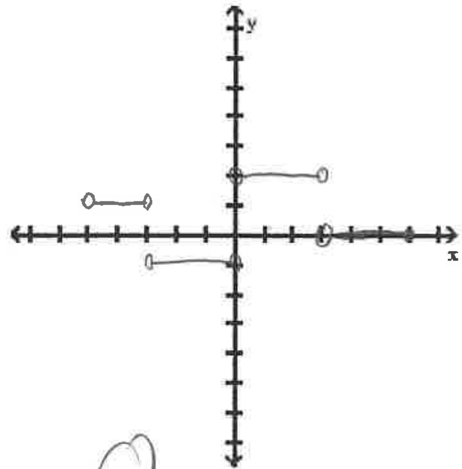
$= \frac{4x^{-1}+3x^{-2}}{x^{-4}} = 4x^3 + 3x^2$

Name:____Date:__________

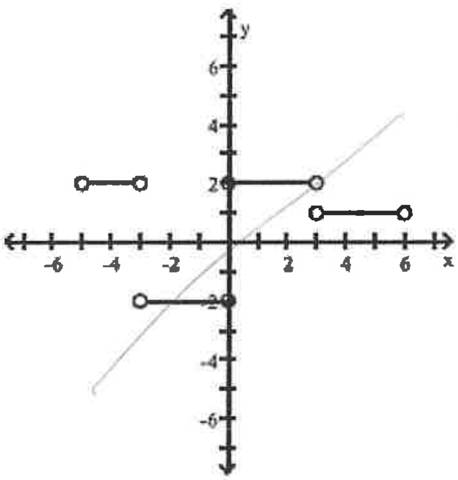
16.

1) The graph of $y = f(x)$ in the accompanying figure is made of line segments joined end to end. Graph the derivative of f.





A)



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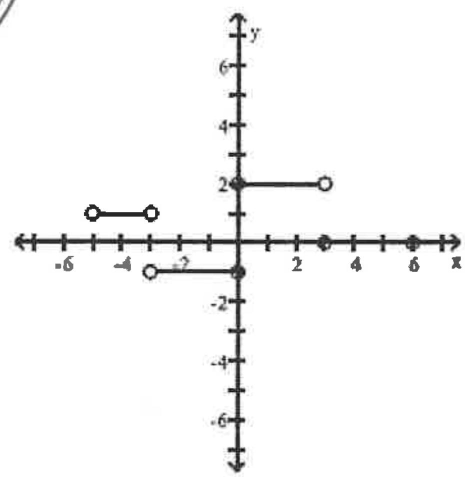

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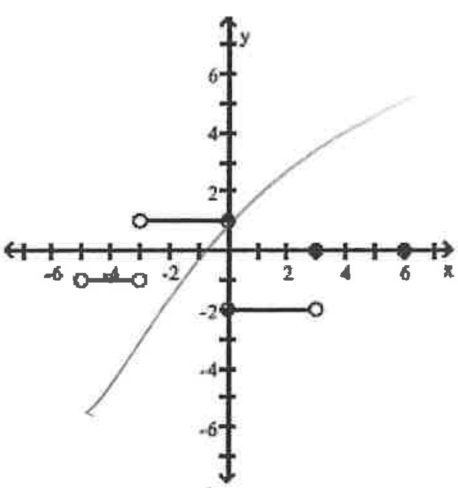

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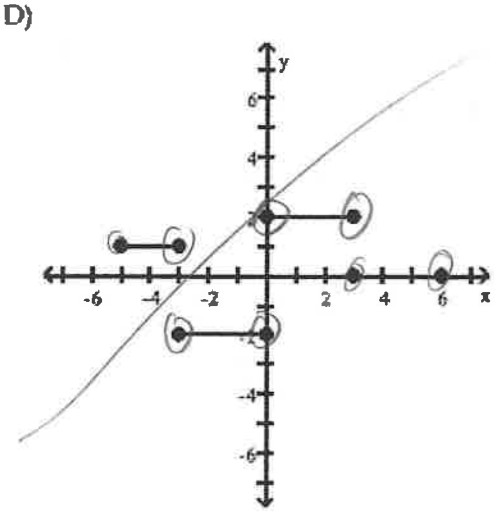


Name:____ Date:__________

17. An equation of the line tangent to the graph of $y = \frac{3x^2-2}{-2x+2}$ at the point (0,-1) is:

Point: $(0,-1)$

slope: $\frac{dy}{dx} = \frac{(-2x+2)(6x)-(3x^2-2)(-2)}{(-2x+2)^2}$ at $x=0$

$\frac{dy}{dx} = \frac{(2)(0)-(-2)(-2)}{(2)^2} = \frac{-4}{4} = -1$ T: $y+1 = -1(x-0)$

Product Rule

18. If $f(x) = x^2 sin(x)$, find $f''(x)$

$f'(x) = x^2\cos(x) + 2x\sin(x)$ Two more products!

$f''(x) = x^2(-\sin(x)) + 2x\cos(x) + 2x\cos(x) + 2\sin(x)$

$= -x^2\sin(x) + 4x\cos(x) + 2\sin(x)$

19. It is known that $f'(x) = 2x^2 - \cos(x)$ and $f\left(\frac{\pi}{2}\right) = 1$. Write the equation of the tangent line to f(x) at $x = \frac{\pi}{2}$.

Point: $(\frac{\pi}{2}, 1)$

slope: $f'(\frac{\pi}{2}) = 2(\frac{\pi}{2})^2 - \cos(\frac{\pi}{2}) = \frac{\pi^2}{2} - 0 = \frac{\pi^2}{2}$

T: $y-1 = \frac{\pi^2}{2}(x - \frac{\pi}{2})$

20. If $g(x) = x^2 + 3x - 1$, identify when g(x) has a horizontal tangent line. Show the work that leads to your answer.

$g'(x) = 0$

$g'(x) = 2x+3 = 0$

$2x+3=0$

$2x=-3$

$\boxed{x = \frac{-3}{2}}$

Name:____Date:__________

21. The table below gives values of f(x), g(x), f'(x), and g'(x) at x=3. Use the table to find the following.

f(x)	g(x)	f'(x)	g'(x)
2	-1	-2	1

a. v'(3) if $v(x) = \frac{f(x)}{g(x)}$

$$v'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$v'(3) = \frac{-1(-2) + 2(1)}{(-1)^2} = \frac{2+2}{1} = \boxed{4}$$

b. v'(3) if $v(x) = 3f(x) + 2g(x) + 2$

$$v'(x) = 3f'(x) + 2g'(x)$$

$$v'(3) = 3(-2) + 2(1) = \boxed{-4}$$

Product

c. v'(3) if $v(x) = e^x g(x) - 3x$

$$v'(x) = e^x g'(x) + e^x g(x) - 3$$

$$v'(3) = e^3(1) + e^3(-1) - 3$$

$$= e^3 - e^3 - 3 = \boxed{-3}$$

22. Find f'(x) if $f(x) = \frac{5}{x^7} - \frac{3}{\sqrt{x}} + 2x$. Then write each term in fraction form.

$$f(x) = 5x^{-7} - 3x^{-1/2} + 2x$$

$$f'(x) = -35x^{-8} + \frac{3}{2}x^{-3/2} + 2$$

$$f'(x) = \frac{-35}{x^8} + \frac{3}{2x^{3/2}} + 2$$

23. If $f(x) = \frac{6}{x^2}\sin(x)$, $f'(x) =$

$$\frac{6}{x^2} = 6x^{-2}$$

Product!

$$f'(x) = \frac{6}{x^2}\cos(x) + (-12x^{-3})\sin(x)$$

$$f'(x) = \frac{6}{x^2}\cos(x) - \frac{12}{x^3}\sin(x)$$

Name:____Date:_________

$f(x) = x^2 + x^{1/2}$ derivative

24. If $f(x) = x^2 + \sqrt{x}$, then the slope of the curve f(x) at x= 1 is

Slope: $f'(x) = 2x + \frac{1}{2}x^{-1/2}$

$f'(1) = 2 + \frac{1}{2} = \boxed{\frac{5}{2}}$

$f(x) = 2x^{-1} + 2x$ derivative

25. If $f(x) = \frac{2}{x} + 2x$, then the slope of the curve f(x) at x= 1 is

$f'(x) = -2x^{-2} + 2 = \frac{-2}{x^2} + 2$

$f'(1) = -2 + 2 = \boxed{0}$

26. Which of the following could be true if $f''(x) = x^2$? You may select more than one answer.

~~a.~~ $f(x) = \frac{3}{2}x^{2/3} - 3$

b. $f(x) = \frac{1}{12}x^4$ $\rightarrow f'(x) = \frac{1}{3}x^3 \rightarrow f''(x) = x^2$ ✓

~~c.~~ $f'''(x) = -\frac{1}{3}x^3$

d. $f'(x) = \frac{1}{3}x^3 + 6$ $\rightarrow f''(x) = x^2$

e. $f'''(x) = 2x$ $\rightarrow f''(x) = x^2$

27. $f(x) = x^3 + 3x$. Find an equation of the line normal to the function at $x = -1$.

Point: $f(-1) = -1 - 3 = -4$

$(-1, -4)$

Slope: $f'(x) = 3x^2 + 3$

$f'(-1) = 3 + 3 = 6$

$\boxed{N: y + 4 = -\frac{1}{6}(x+1)}$

28. If $y = 2x^2 - 2\sqrt{x}$ then $\frac{dy}{dx} =$

$y = 2x^2 - 2x^{1/2}$

$\frac{dy}{dx} = 4x - x^{-1/2} = \boxed{4x - \frac{1}{\sqrt{x}}}$

Name:____Date:_________

29. An equation of the line tangent to the graph of $y = 2x^2 - 2x$ at the point (2, 4) is:

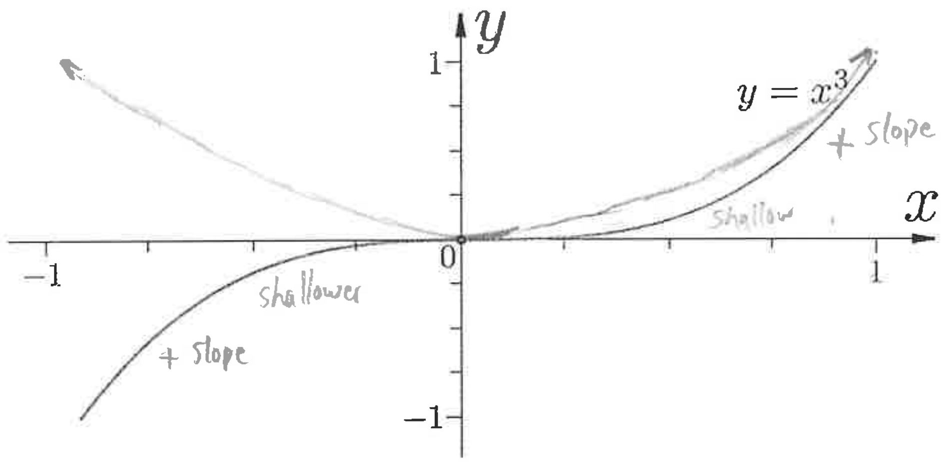
Point: (2,4)

$\frac{dy}{dx} = 4x-2$

at $x=2 \rightarrow 4(2)-2=6$

T: $y-4=6(x-2)$

30. Sketch the graph of the derivative of the following function on the same coordinate plane.



31. The given limit is a derivative. What function is it a derivative of and at what point? Then evaluate the derivative.

$$\lim_{x \to -3} \frac{\frac{3}{x}+1}{x+3}$$

$f(x) = \frac{3}{x} = 3x^{-1}$

$a=-3$

$f'(x) = -3x^{-2}$

$f'(-3) = \frac{-3}{(-3)^2} = \boxed{-\frac{1}{3}}$

Name:____Date:__________

32. The table below gives the approximate distance traveled by a plane after t minutes for $0 \le t \le 10$. (Include units in both of your answers)

Distance (miles)	0	10	19	27	35	46	53	64	75	87	100
Time(minutes)	0	1	2	3	4	5	6	7	8	9	10

a. What is the plane's average rate of change for $1 \le t \le 9$? (include units—can leave unsimplified)

$\frac{d(9)-d(1)}{9-1} = \frac{87-10 \text{ mi}}{9-1 \text{ min}} = \frac{77}{8}$ mi/min

b. Approximate the plane's instantaneous rate of change at t=5.3 seconds. (include units)

$\frac{d(6)-d(5)}{6-5} = \frac{53-46 \text{ mi}}{1 \text{ min}} = 7$ mi/min

33. Which of the following is true about $h(x) = \begin{cases} 3x + \cos(x), & x \le 0 \\ e^x + 2x, & x > 0 \end{cases}$

a. h(x) is continuous and differentiable at x=0
b. h(x) is continuous but not differentiable at x=0
c. h(x) is neither continuous nor differentiable at x=0
d. h(x) is continuous but not differentiable at x=0

continuity:

$3x + \cos(x) = e^x + 2x$ at $x=0$

$0 + 1 = e^0 + 0$

$1 = 1$ ✓

differentiability:

$3 - \sin(x) = e^x + 2$ at $x = 0$

$3 - 0 = 1 + 2$

$3 = 3$ ✓