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P 2016

# Unit 3: Advanced Derivatives + Motion I V2

1. Determine the value of f '(1) if $f(x) = x(3-2x)^4$.

$f'(x) = x \cdot 4(3-2x)^3(-2) + (3-2x)^4$ | $x=1$

$f'(1) = 4(1)^3(-2) + 1^4$

$f'(1) = -8+1 = -7$

2. If $f(x) = (4-x)^4$, then the third derivative of f(x) at x=1 is

a. -12   b. -72   c. 36   d. 12   e. 72

$f'(x) = 4(4-x)^3(-1) = -4(4-x)^3$

$f''(x) = -12(4-x)^2(-1) = 12(4-x)^2$

$f'''(x) = 24(4-x)(-1) = -24(4-x)$

$f'''(1) = -24(4-1) = -24(3) = -72$

3. If $y = \sin(\pi x)$, find the first derivative y' at $x = \frac{1}{3}$

$y' = \cos(\pi x)\pi$   $x = \frac{1}{3}$

$y' = \cos(\pi/3)\pi = \frac{1}{2} \cdot \pi = \frac{\pi}{2}$

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4. Find the derivative of $f(x) = \frac{1}{\sqrt[3]{2-x^2}}$

a. $\frac{4x}{3\sqrt[3]{(2-x^2)^4}}$ b. $\frac{-2x}{3\sqrt[3]{(2-x^2)^4}}$ c. $\frac{-4x}{3\sqrt[3]{(2-x^2)^4}}$ d. $\frac{-1}{3\sqrt[3]{(2-x^2)^4}}$ e. $\frac{2x}{3\sqrt[3]{(2-x^2)^4}}$

$f(x) = \frac{1}{\sqrt[3]{2-x^2}} = \frac{1}{(2-x^2)^{1/3}} = (2-x^2)^{-1/3}$

$f'(x) = -\frac{1}{3}(2-x^2)^{-4/3} \cdot (-2x) = \frac{-1}{3} \cdot \frac{1}{(2-x^2)^{4/3}} \cdot (-2x)$

$= \frac{-1}{3} \frac{1}{\sqrt[3]{(2-x^2)^4}} \cdot (-2x) = \frac{2x}{3\sqrt[3]{(2-x^2)^4}}$

5. $\frac{d}{dx} cos^2(2x^4)$

a. $-8x^3 \sin(x^4)\cos(x^4)$ b. $-16x^3 \sin(x^4)\cos(x^4)$

c. $-16x^3 \cos(2x^4)\sin(2x^4)$

d. $2\cos(2x^4)$ e. $-\sin(2x^4)$

$\cos^2(2x^4) = [\cos(2x^4)]^2$

Chain Rule: $2\cos(2x^4) \cdot (-\sin(2x^4))(8x^3) = -16x^3 \sin(2x^4)\cos(2x^4)$

6. $\lim\limits_{h \to 0} \frac{\ln(e+h)-0}{h}$ is ← limit def'n of deriv.
Says: take deriv of $\ln(x)$ at $x=e$

(A) f ' (e), where f(x)=lnx

(B) f ' (e), where $f(x) = \frac{\ln x}{x}$

(C) f ' ~~(1)~~, where f(x)=lnx

(D) f ' ~~(1)~~, where f(x)=ln (x+e)

(E) f ' ~~(0)~~, where f(x)=lnx

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Chain Rule!

7. Determine f '(4) if $f(x) = e^{\frac{-x}{4}}$

a. $\frac{-1}{4}e^{-1}$ b. $e^{-\frac{5}{3}x}$ c. ln(1/4) d. $\frac{-1}{4}e$ e. $\frac{3}{4}e^{-\frac{1}{4}x}$

$f'(x) = e^{\frac{-x}{4}} \cdot \frac{-1}{4}$ ooo Deriv of $-\frac{x}{4} \rightarrow \frac{-1}{4}$

$f'(4) = e^{-\frac{4}{4}} \cdot \frac{-1}{4} = e^{-1} \cdot \frac{-1}{4}$ $\frac{1}{e}\left(\frac{-1}{4}\right) =$

8. If $y = 4^{1-x^2}$, then $\frac{dy}{dx} =$

Chain Rule!

$\frac{dy}{dx} = 4^{1-x^2}\ln(4) \cdot (-2x)$

9. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e$ is

→ derivative ... → chain!

a. $\frac{2}{e^2}$ b. $\frac{1}{e^2}$ c. $\frac{-1}{e^2}$ d. $\frac{2}{e}$ e. $\frac{1}{e^3}$

$\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x = \frac{2x}{x^2} = \frac{2}{x}$ at $x=e$

$\frac{2}{e}$

10. An equation of the line tangent to the graph of $y = \frac{3x^2-2}{-2x+2}$ at the point (0,-1) is:

Point: Given $(0,-1)$

slope: $\frac{dy}{dx} = \frac{(-2x+2)(6x) - (3x^2-2)(-2)}{(-2x+2)^2}$ $x = 0$

$\frac{dy}{dx} = \frac{(2)(0) - (-2)(-2)}{(2)^2} = \frac{-4}{4} = -1$

$\boxed{y+1 = -1(x-0)}$

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11. If $f(x) = x^2 \underset{\text{Product}}{sin(x)}$, find $f''(x)$

$f'(x) = x^2 \cos(x) + 2x \sin(x)$ Product again

$f''(x) = x^2(-\sin(x)) + 2x\cos(x) + 2x\cos(x) + 2\sin(x)$

$f''(x) = -x^2\sin(x) + 4x\cos(x) + 2\sin(x)$

12. Which of the following is $\frac{d}{dx}\tan^{-1}(2x)$ ? Chain

a. $\frac{2}{1+2x^2}$ b. $\frac{1}{1+4x^2}$ c. $\frac{2}{\sqrt{1-4x^2}}$ d. $\frac{2}{1+4x^2}$ e. $\frac{2}{\sqrt{1-4x^2}}$

$\frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$

13. $\frac{d}{dx} arc\cos(x^4) =$ Chain $\frac{-1}{\sqrt{1-(x^4)^2}} \cdot 4x^3 = \frac{-4x^3}{\sqrt{1-(x^8)}}$

a. $-\frac{1}{\sqrt{1-x^2}}$ b. $\frac{4x^3}{\sqrt{1-x^4}}$ c. $-\frac{4x^3}{\sqrt{1-x^8}}$ d. $\frac{1}{1+x^8}$ e. $\frac{3x^3}{\sqrt{1+x^4}}$

14. The table below gives values of f(x), g(x), f'(x), and g'(x) at x=3. Use the table to find v'(3) if $v(x) = \frac{f(x)}{g(x)}$.

| f(x) | g(x) | f'(x) | g'(x) |
|---|---|---|---|
| 2 | -1 | -2 | 1 |

Quotient: $v'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \frac{(-1)(-2) - (2)(1)}{(-1)^2} = \frac{2-2}{1} = 0$

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15. An object moves along the x-axis in such a way that its x-coordinate at any time is given by the equation $x(t) = 6t^2 - t^3$. What is the object's position at the instant the acceleration is zero?

Need position when $a(t) = 0$

$x(t) = 6t^2 - t^3 \longrightarrow x(2) = 6(2)^2 - 2^3 = 6(4) - 8 = 16$

$v(t) = 12t - 3t^2$

$a(t) = 12 - 6t$

$0 = 12 - 6t$

$t = 2$

16. The velocity of a particle moving along a straight line at any time t is given by $v(t) = 2t^3 + 4t$. What is the acceleration of the particle when t=2?

$v(t) = 2t^3 + 4t$

deriv $a(t) = 6t^2 + 4$

$a(2) = 6(2)^2 + 4 = 28$

17. A particle moves along a line so that its position at any time t≥0 is given by $s(t) = 2 + 5t - 2t^2$. When is the particle moving right?

$v(t)$ is +

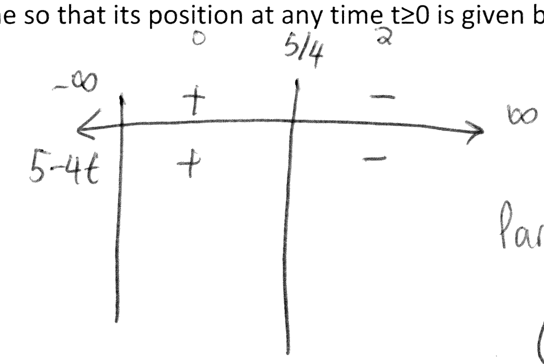
$s(t) = 2 + 5t - 2t^2$

$v(t) = 5 - 4t$

$5 - 4t = 0$

$-4t = -5$

$t = \frac{5}{4}$




Particle is moving right on the interval $(-\infty, 5/4)$ b/c $v(t)$ is +

18. If $f(x) = \sin(x)$ and $g(x) = e^x$ find the derivative of f(g(x)) at x=0.

$f(g(x)) = \sin(e^x)$ Chain Rule

Deriv: $\cos(e^x)e^x$ $x = 0$

$\cos(e^0) \cdot e^0$

$\cos(1) \cdot 1 = \cos(1)$

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**Calculator Active**

19. Oil is pumped into and out of a storage container at refinery. The amount of oil in the oil container is given by the equation $a(t) = \sin(x^2 + 1) + e^{\sin(x)} + 1$. a measures the amount of oil in thousands of gallons while t measures time in days.

(a) What is the initial amount of oil in the container?

$a(0) = \sin(1) + e^{\sin(0)} + 1$ thousands of gallons

→ rate = deriv

(b) How quickly is the amount of oil changing on day 5? Does this mean the amount of oil in the container is increasing or decreasing? Explain.

$a'(5) \Rightarrow$ Calculator

6.578 thousands of gallons/day

The amount of oil in the container is increasing b/c the rate $a'(t)$ is +

20. If a particle's acceleration is 0, is the particle's velocity 0 too? Explain your answer.

No, a particle could have a velocity but not be speeding up or slowing down.

E.g. $v(t) = 5$

$a(t) = 0$

Excellence Points

i. Find the equation of the normal line to the curve y=2sin(πx) when x is ¼.

Point: $y = 2\sin(\pi/4) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$ $(1/4, \sqrt{2})$

Slope: $\frac{dy}{dx} = 2\cos(\pi x)\pi$ $x = \frac{1}{4}$

$2\cos(\pi/4)\pi = 2\frac{\sqrt{2}}{2} \cdot \pi = \sqrt{2}\pi$

N: $y - \sqrt{2} = \frac{-1}{\sqrt{2}\pi}\left(x - \frac{1}{4}\right)$

ii. $\lim_{x\to\infty} \frac{(x^3+4)}{3^x} =$ $\frac{\text{small}}{\text{BIG}} = 0$

Name:
Score: Answer

**Unit 3 Pre-Test**

1. Find the equation of the tangent line to $f(x) = e^{\frac{x}{3}}$ at x=6.

a. $y - e = 2e(x-6)$
b. $y - e^2 = e^2(x-6)$
c. $y - e^2 = 2e^2(x-6)$ no
d. $y - e^2 = \frac{e^2}{3}(x-6)$

Point: $f(6) = e^{\frac{6}{3}} = e^2$

$(6, e^2)$

Slope: $f'(x) = e^{\frac{x}{3}} \cdot \frac{1}{3}$

$f'(6) = e^2 \cdot \frac{1}{3} = \frac{e^2}{3}$

$y - e^2 = \frac{e^2}{3}(x-6)$

2. Given $f(x) = x^2\sin(x)$ what is f'(x)?

$f'(x) = x^2\cos(x) + 2x\sin(x)$

3. Given $f(x) = x(2x^2 - x)^3$, what is f'(1)?

$f'(x) = x \cdot 3(2x^2 - x)^2(4x - 1) + (2x^2 - x)^3$

$f'(1) = 1 \cdot 3(2-1)^2(4-1) + (2-1)^3$

$= 1 \cdot 3 \cdot (1) \cdot 3 + (1)^3$

$= 9 + 1 = 10$

**Review**

Product Rule:

1. If $f(x) = x^2 sin(x)$, find $f''(x)$

$f'(x) = x^2\cos(x) + 2x\sin(x)$

$f''(x) = x^2(-\sin(x)) + 2x\cos(x) + 2x\cos(x) + 2\sin(x)$

$f''(x) = -x^2\sin(x) + 4x\cos(x) + 2\sin(x)$

Quotient Rule

2. The equation of the line tangent to the graph of $y = \frac{3x^2-4}{-2x+2}$ at the point (0,-2) is:

Point: $(0,-2)$

slope: $\frac{dy}{dx} = \frac{(-2x+2)(6x) - (3x^2-4)(-2)}{(-2x+2)^2}$ at $x=0$ $y + 2 = -2(x-0)$

$\frac{dy}{dx} = \frac{0 - (-4)(-2)}{(2)^2} = -\frac{8}{4} = -2$

Name:
Score:

Chain Rule:

3. If $y = (3x+1)^4$ , what is the $\frac{d^3y}{dx^3}$ at x=1?

$$\frac{dy}{dx} = 4(3x+1)^3(3) = 12(3x+1)^3$$
$$\frac{d^2y}{dx^2} = 36(3x+1)^2 \cdot 3 = 108(3x+1)^2$$
$$\frac{d^3y}{dx^3} = 216(3x+1)(3) \quad \text{at } x=1$$
$$= 216(4)(3) = 2592$$

4. If $f(x) = x^2 - 1$ and $g(x) = \ln(x)$ find the derivative of f(g(x)) at x=1.

$$f(g(x)) = [\ln(x)]^2 - 1$$
Derivative: $2\ln(x) \cdot \frac{1}{x}$
$$x=1 \rightarrow 2\ln(1) \cdot \frac{1}{1}$$
$$0$$

Combine them:

5. If $f(x) = 2^x sin(2x)$, find $f'(x)$

$$f'(x) = 2^x \cos(2x) \cdot 2 + 2^x \ln(2) \sin(2x)$$

ALL DERIVATIVE RULES ARE FAIR GAME:

6. If $f(x) = \sin^{-1}(2x)$ find $f'(x)$

$$f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

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Unit 2 – Sprint Logs 1

Correct:__________

# A

| | | |
|---|---|---|
| 1 | $\log(10)$ | 1 |
| 2 | $\log(1000)$ | 3 |
| 3 | $\log(\frac{1}{10})$ | -1 |
| 4 | $\log_3(27)$ | 3 |
| 5 | $\log_3(\frac{1}{9})$ | -2 |
| 6 | $\log_3(81)$ | 4 |
| 7 | $\log_7(\frac{1}{49})$ | -2 |
| 8 | $\log_7(343)$ | 3 |
| 9 | $\log_7(1)$ | 0 |
| 10 | $\log_4(64)$ | 3 |
| 11 | $\log_4(\frac{1}{16})$ | -2 |
| 12 | $\log_4(\frac{1}{64})$ | -3 |
| 13 | $\log_5(25)$ | 2 |
| 14 | $\log_5(\frac{1}{5})$ | -1 |
| 15 | $\log_5(1)$ | 0 |

| | | |
|---|---|---|
| 16 | $\log_{\frac{1}{7}}(\frac{1}{49})$ | 2 |
| 17 | $\log_{\frac{1}{7}}(\frac{1}{7})$ | 1 |
| 18 | $\log_{\frac{1}{7}}(49)$ | -2 |
| 19 | $\log_{\frac{1}{3}}(81)$ | -4 |
| 20 | $\log_{\frac{1}{3}}(1)$ | 0 |
| 21 | $\log_{\frac{1}{10}}(1000)$ | -3 |
| 22 | $\log_{\frac{1}{10}}(\frac{1}{100})$ | 2 |
| 23 | $\log_{\frac{1}{7}}(1)$ | 0 |
| 24 | $\log_{\frac{1}{7}}(7^{-3})$ | 3 |
| 25 | $\log_{\frac{1}{3}}(3^6)$ | -6 |
| 26 | $\log_{\frac{1}{10}}(100^2)$ | -4 |
| 27 | $\log_{\frac{1}{10}}(100^3)$ | -6 |
| 28 | $\log_x(x^3)$ | 3 |
| 29 | $\log_x(\frac{1}{x})$ | -1 |
| 30 | $\log_{x^2}(x^4)$ | 2 |

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Unit 2 – Sprint Logs 1

Correct:___________

# B

| | | |
|---|---|---|
| 1 | $\log(100)$ | 2 |
| 2 | $\log(\frac{1}{100})$ | -2 |
| 3 | $\log(1)$ | 0 |
| 4 | $\log_2(16)$ | 4 |
| 5 | $\log_2(\frac{1}{4})$ | -2 |
| 6 | $\log_2(8)$ | 3 |
| 7 | $\log_5(\frac{1}{25})$ | -2 |
| 8 | $\log_5(25)$ | 2 |
| 9 | $\log_5(1)$ | 0 |
| 10 | $\log_4(64)$ | 3 |
| 11 | $\log_4(\frac{1}{16})$ | -2 |
| 12 | $\log_4(\frac{1}{64})$ | -3 |
| 13 | $\log_6(36)$ | 2 |
| 14 | $\log_6(6)$ | 1 |
| 15 | $\log_6(\frac{1}{36})$ | -2 |

| | | |
|---|---|---|
| 16 | $\log_{\frac{1}{5}}(\frac{1}{5})$ | 1 |
| 17 | $\log_{\frac{1}{5}}(\frac{1}{25})$ | 2 |
| 18 | $\log_{\frac{1}{5}}(5)$ | -1 |
| 19 | $\log_{\frac{1}{5}}(125)$ | -3 |
| 20 | $\log_{\frac{1}{2}}(4)$ | -2 |
| 21 | $\log_{\frac{1}{2}}(1)$ | 0 |
| 22 | $\log_{\frac{1}{10}}(100)$ | -2 |
| 23 | $\log_{\frac{1}{10}}(\frac{1}{100})$ | 2 |
| 24 | $\log_{\frac{1}{5}}(5^2)$ | -2 |
| 25 | $\log_{\frac{1}{5}}(5^{-2})$ | +2 |
| 26 | $\log_{\frac{1}{5}}(5^4)$ | -4 |
| 27 | $\log_{\frac{1}{10}}(1000^2)$ | -6 |
| 28 | $\log_x(x^5)$ | 5 |
| 29 | $\log_x(\frac{1}{x})$ | -1 |
| 30 | $\log_{x^2}(x^6)$ | 3 |