Station 1

Simplify each exponential expression.

~~$\frac{19}{4}$ = Remainder of 3 $i^3 = -i$~~

24) i^{19}

A) -i B) -1 C) i D) 1

25) i^{21} Remainder of 1

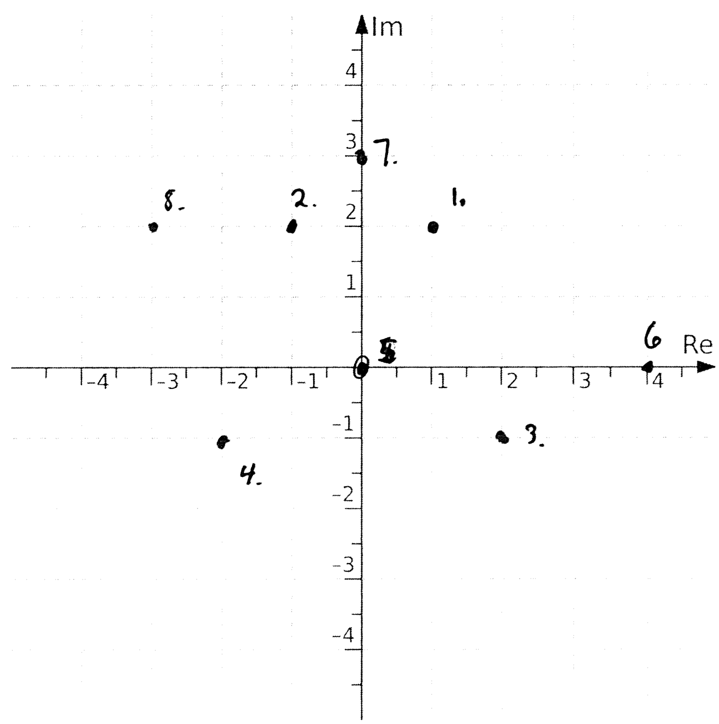
A) i B) -1 C) 1 D) -i

26) i^{14} Remainder of 2 $i^2 = -1$

A) i B) -1 C) 1 D) -i

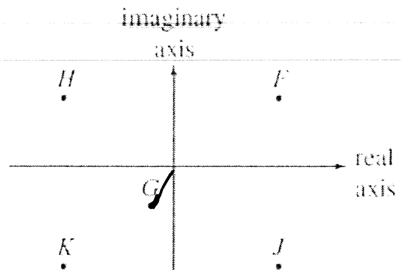
Graph the following numbers on the axis at the right. Label them after you graph them. You do not need to draw an arrow to the point.

1. $1 + 2i$
2. $-1 + 2i$
3. $2 - i$
4. $-2 - i$
5. 0
6. 4
7. $3i$
8. $-3 + 2i$



48. In a complex plane, the vertical axis is the *imaginary axis* and the horizontal axis is the *real axis*. Within the complex plane, a complex number $a + bi$ is comparable to the point (a, b) in the standard (x, y) coordinate plane. $\sqrt{a^2 + b^2}$ is the modulus of the complex point $a + bi$. Which of the complex numbers F, G, H, J, and K below has the smallest modulus?

modulus = distance from origin



F. F
G. G
H. H
J. J
K. K

Find the absolute value of each complex number.

1) $|7 - i|$ $\sqrt{7^2 + (1)^2} = \sqrt{50} = 5\sqrt{2}$

2) $|-5 - 5i|$ $\sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

3) $|-2 + 4i| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$

4) $|3 - 6i| = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$

5) $|10 - 2i| = \sqrt{10^2 + 2^2} = \sqrt{104} = 2\sqrt{26}$

6) $|-4 - 8i| = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$

Station 2

Find the product.	Factor
2. $(z-4)(z+4)$ z^2-16	6. z^2-144 $(z+12)(z-12)$
3. $(z+3i)(z-3i)$ remember our rule! You can foil to check z^2+9	7. y^2+16 $(y+4i)(y-4i)$
4. $(z+\sqrt{13})(z-\sqrt{13})$ z^2-13	8. z^2+15 $(z+\sqrt{15}i)(z-\sqrt{15}i)$
5. $(z+\sqrt{5}i)(z-\sqrt{5}i)$ z^2+5	9. t^2-9i $(t+3\sqrt{i})(t-3\sqrt{i})$
	10. z^2+25i $(z+5\sqrt{i}\,i)(z-5\sqrt{i}\,i)$

Examples

11. Solve each equation, and state the solutions.

a. $x^2+64=0$

$(x+8i)(x-8i)=0$

$x=8i$

$x=-8i$

b. $x^2+10x+25=0$

$(x+5)(x+5)=0$

$x=-5$

12. Write the left side of each equation as a product of linear factors, and state the solutions.

a. $x^3 - 125 = 0$

$(x^2+5x+25)$

$(x-5)(x^2+5x+25)=0$ $\quad x=\dfrac{-5\pm\sqrt{25-4(1)25}}{2(1)}=\dfrac{-5\pm\sqrt{-75}}{2}=\dfrac{-5\pm i\sqrt{75}}{2}$

$(x-5)\left(x-\left[\dfrac{-5+i\sqrt{75}}{2}\right]\right)\left(x-\left[\dfrac{-5-i\sqrt{75}}{2}\right]\right)=0$

Solutions:
$x=5$
$x=\dfrac{-5\pm i\sqrt{75}}{2}$

b. $x^3 + 8 = 0$

$(x+2)(x^2-2x+4)=0$ $\quad x=\dfrac{+2\pm\sqrt{4-4(1)(4)}}{2}=\dfrac{+2\pm\sqrt{-12}}{2}=\dfrac{+2\pm i\sqrt{12}}{2}$

$(x+2)\left(x-\left[\dfrac{+2+i\sqrt{12}}{2}\right]\right)\left(x-\left[\dfrac{+2-i\sqrt{12}}{2}\right]\right)=0$

solution: $x=-2$
$x=\dfrac{+2\pm i\sqrt{12}}{2}$

c. $x^4 + 6x^2 + 8 = 0$ $\quad u=x^2$

$u^2+6u+8=0$

$(u+4)(u+2)=0$

$(x^2+4)(x^2+2)=0$

$(x+2i)(x-2i)(x+\sqrt{2}i)(x-\sqrt{2}i)=0$

Solutions: $x=\pm 2i$
$x=\pm\sqrt{2}i$

d. $x^4 + 9x^2 + 8 = 0$ $\quad u=x^2$

$u^2+9u+8=0$

$(u+8)(u+1)=0$

$(x^2+8)(x^2+1)=0$

$(x+\sqrt{8}i)(x-\sqrt{8}i)(x+i)(x-i)=0$

Solutions:

$x=\pm\sqrt{8}i$

$x=\pm i$

Station 3

13. Explain how Pascal's triangle allows you to compute the coefficient of x^3y^2 when $(x-y)^5$ is expanded. What is that coefficient?

Either this:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$\boxed{10}$

Or this:

we need 5th row, 2nd item because y^2.

C(5,2) on calc: 10

14. Explain how Pascal's triangle allows you to compute the coefficient of x^5y when $(x+y)^6$ is expanded. What is that coefficient?

Either this:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$\boxed{6}$

Or this:

C(6,1) on calc: 6

Simplify each expression to the form a+bi.

15. (1-3i)+(2+i)(1+i)

$1-3i+2+2i+i+i^2$

$\cancel{1}-3i+2+2i+i\cancel{-1}$

$\boxed{2}$

16. $(1+i)^3-(1-i)^3$ Use Pascal's Triangle

1
1 1
1 2 1
1 3 3 1

$1\cdot1^3+3\cdot1^2i+3\cdot1\cdot i^2+1\cdot i^3-[1\cdot1^3+3\cdot1^2(-i)+3\cdot1\cdot(-i)^2+1\cdot(-i)^3]$

$1+3i+3i^2+i^3-[1+3(-i)+3(-i)^2+(-i)^3]$

$1+3i+3i^2+i^3-[1-3i+3i^2-i^3]$

$\cancel{1}+3i+\cancel{3i^2}+i^3-\cancel{1}+3i-\cancel{3i^2}+i^3$

$6i+2i^3$

$6i-2i$ $\boxed{0+4i}$

~~17. $(2+i)^4-(2-i)^4$~~

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$1\cdot 2^4+4\cdot 2^3\cdot i+6\cdot 2^2 i^2+4\cdot 2\cdot i^3+i^4-[1\cdot 2^4+4\cdot 2^3(-i)+6(2)^2(-i)^2+4(2)(-i)^3+(-i)^4]$

$16+32i+24i^2+8i^3+i^4-[16+32(-i)+24(-i)^2+8(-i)^3+(-i)^4]$

$16+32i+24i^2+8i^3+i^4-[16-32i+24i^2-8i^3+i^4]$

$\cancel{16}+32i+\cancel{24i^2}+8i^3\cancel{+i^4}-\cancel{16}+32i-\cancel{24i^2}+8i^3\cancel{-i^4}$

$64i+16i^3=64i-16i=48i$

18. Consider the expansion of $(a+b)^7$. Determine the coefficients for the terms with the powers of a and b shown.

a. a^2b^5

$C(7,5)=21$

b. a^6b

$C(7,1)=7$

c. b^7

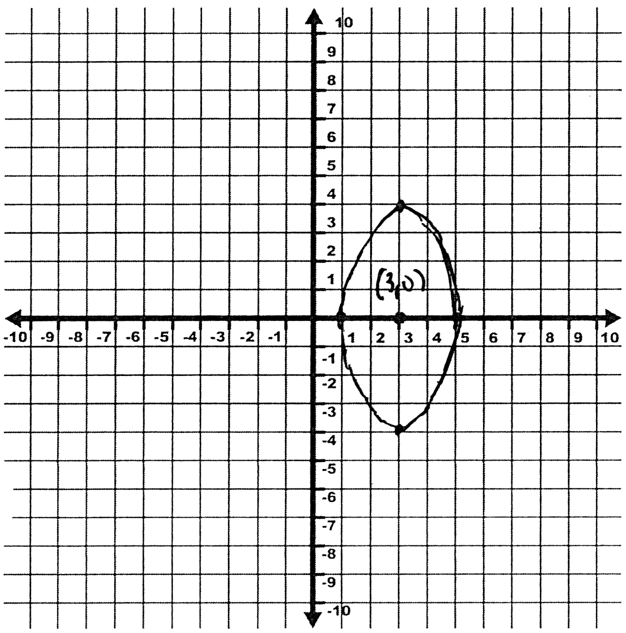
$C(7,7)=1$

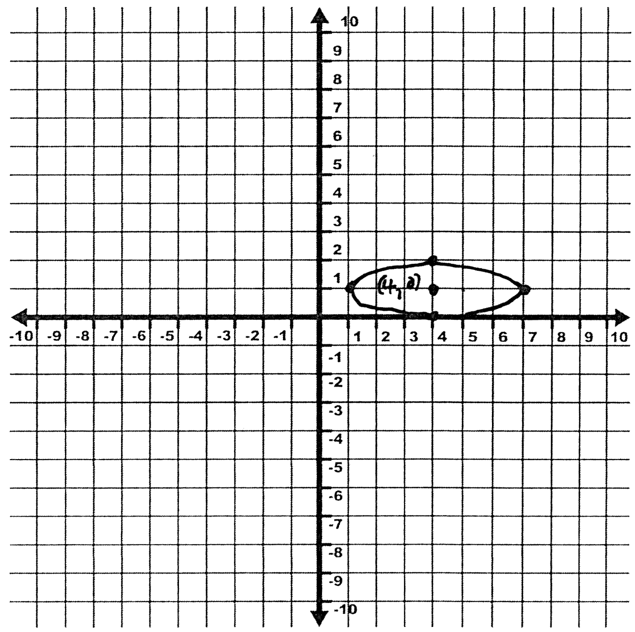
Station 4

Sketch the graphs of each equation.

$$\frac{(x-3)^2}{4}+\frac{y^2}{16}=1$$

$$\frac{(x-4)^2}{9}+(y-1)^2=1$$



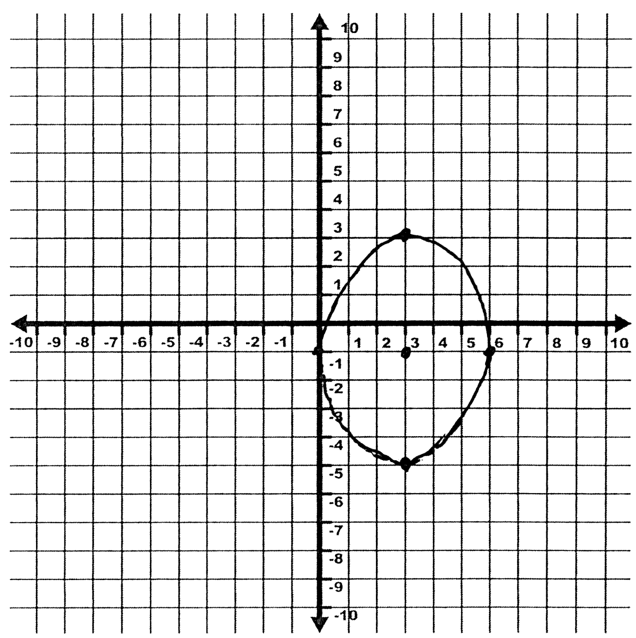


$$\frac{(x-3)^2}{9}+\frac{(y+1)^2}{16}=1$$

C: (3,-1)

semi-major radius: 4 in y-dir

semi-minor radius: 3 in x-dir



$$\frac{(x+2)^2}{49} + \frac{(y-1)^2}{9} = 1$$

a. What is the center?

$(-2, 1)$

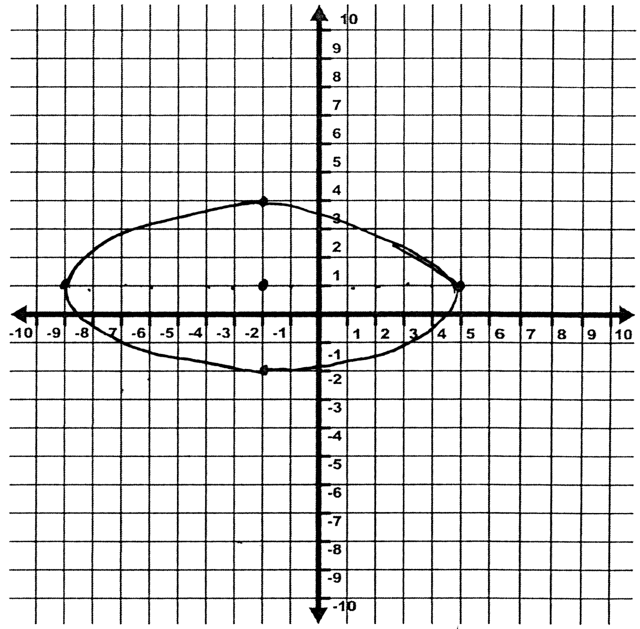
b. What is the semi-major axis?

7 in the x-direction

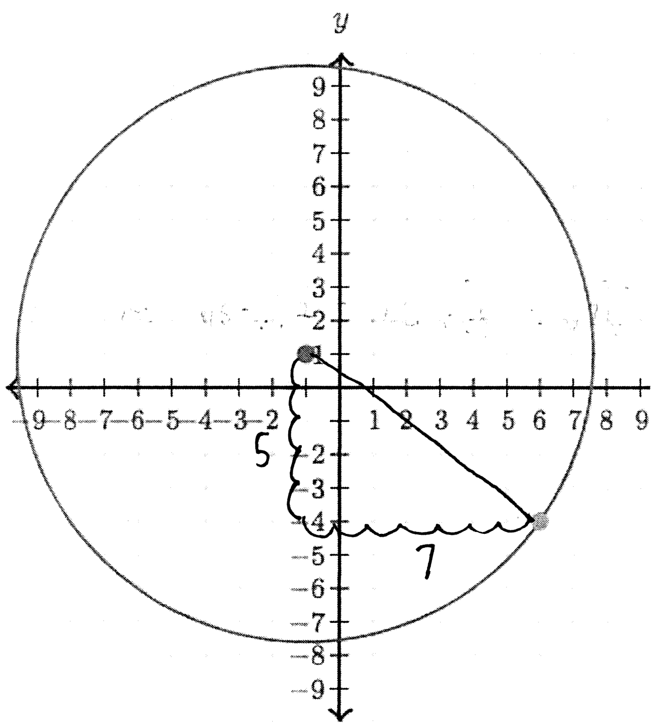
c. What is the semi-minor axis?

3 in the y-direction

d. Graph the ellipse.



Write the equation of the circle graphed below.



Center: $(-1, 1)$

$r = \sqrt{5^2 + 7^2} = \sqrt{25+49} = \sqrt{74}$

$$\boxed{(x+1)^2 + (y-1)^2 = 74}$$

Station 5

1. Given the function $f(x) = (x+1)^2(x-1)(x+4)^3$ determine the intervals on which the function is positive and negative.

	+	-4 −	-1 −	1 +
$(x+1)^2$	+	+	+	+
$(x-1)$	−	−	−	+
$(x+4)^3$	−	+	+	+

$+$ intervals: $(-\infty,-4)\ (1,\infty)$

$-$ intervals: $(-4,-1)\ (-1,1)$

2. Consider the cubic polynomial p given $p(t) = t^3 - 125$.

a. Find a real number zero/root to the polynomial by factoring.

$p(t) = (t-5)(t^2+5t+25)$

$t=5$

b. Write $p(t)$ as a product of three linear terms. Then state ALL of the zeros.

$t = \dfrac{-5 \pm \sqrt{25-4(1)(25)}}{2} = \dfrac{-5 \pm \sqrt{-75}}{2} = \dfrac{-5 \pm \sqrt{75}i}{2} = \dfrac{-5 \pm 5\sqrt{3}i}{2}$

$(t-5)\left(t-\left[\dfrac{-5+5\sqrt{3}i}{2}\right]\right)\left(t-\left[\dfrac{-5-5\sqrt{3}i}{2}\right]\right)=0$

3. Find all of the factors (real and complex) to the polynomial $r(x) = x^4 - 81$

$u = x^2$

$r = u^2 - 81$

$r = (u-9)(u+9)$

$r(x) = (x^2-9)(x^2+9)$

$r(x) = (x-3)(x+3)(x+3i)(x-3i)$

4. In the expansion of the polynomial $(x+3y)^3$, what is the coefficient in front of the term xy^2?

$x^3 + 3x^2(3y) + 3x(3y)^2 + (3y)^3$

$x^3 + 9x^2y + 27xy^2 + 27y$

$\boxed{27}$

5. In the expansion of the polynomial $(2x+4y)^4$, what is the coefficient in front of the term x^2y^2?

$(2x)^4 + 4(2x)^3(4y) + 6(2x)^2(4y)^2 + 4(2x)(4y)^3 + (4y)^4$

$+ 6 \cdot 4x^2 \cdot 16y^2$

$+ 384x^2y^2$

$\boxed{384}$