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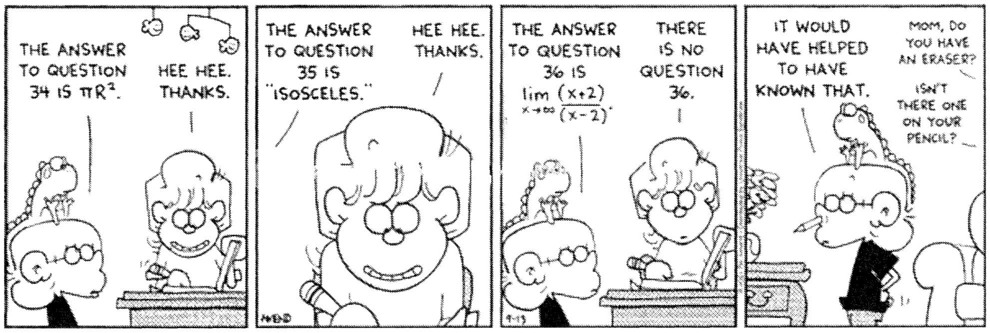
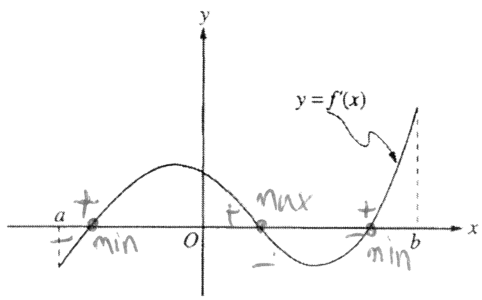
AP Calculus AB Work hard. Be nice.

# Unit 4: Applied Derivatives Practice




NonCalc:




The graph of $f'$, the derivative of $f$, is shown in the figure above. Which of the following describes all relative extrema of $f$ on the open interval $(a, b)$?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

max when $f'(x)$: $+ \to -$
min when $f'(x)$: $- \to +$

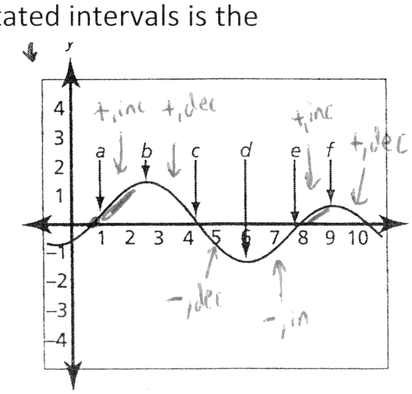
derivative

2. The graph of g'(x) is shown on the graph to the right. For which of the stated intervals is the function g(x) both increasing and concave up?

a. a<x<b
b. e<x<f
c. a<x<b and e<x<f
d. a<x<c and e<x<h
e. a<x<b and d<x<f

→ g'(x) is + → g'(x) is inc

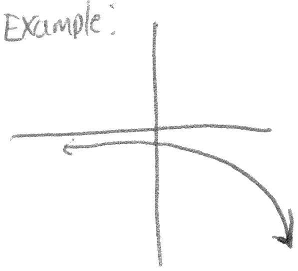
need both



below x-axis, decreasing, CCD

3. The function f has the property that $f(x)$, $f'(x)$ and $f''(x)$ are negative for all real values x. Which of the following could be the graph of $f(x)$?

Example:



4. Which of the following conditions would enable you to conclude that the graph of f has a local maximum at x=c?

- ~~a.~~ $f'(c)=0$ not enough
- ~~b.~~ $f'(c)$ changes from increasing to decreasing no; if f(x) changed inc to dec
- ~~c.~~ $f''(c)>0$ CCU: no
- ~~d.~~ The sign of $f''(x)$ changes at x=c no; POI
- e. $f'(c)=0$ and $f''(c)<0$

$f'(x)=0$ ✓ and $f''(x)<0$ CCD

5. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

f(x) Inc if $f'(x)$ is +:

a. $-3<x<1$
b. $-1<x<1$
c. $x<-3$ or $x>1$
d. $x<-1$ or $x>3$
e. all real numbers

$f'(x)=3x^2+6x-9$

$f'(x)=3(x^2+2x-3)$

$0=3(x+3)(x-1)$

$x=-3 \quad x=1$

| | $x<-3$ | $-3<x<1$ | $x>1$ |
|---|---|---|---|
| | + | − | + |
| 3 | + | + | + |
| $(x+3)$ | − | + | + |
| $(x-1)$ | − | − | + |

$f'(x)$: + $\quad x<-3$, $x>1$

6. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

↳ $y''$ is −

a. $X<0$
b. $X>0$
c. $X<-2$ or $x>-2/3$
d. $X<2/3$ or $x>2$
e. $2/3 <x <2$

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48$

$y'' = 6(6x^2 - 16x + 8)$ oops! better

$y'' = 12(3x^2 - 8x + 4)$

$12(3x-2)(x-2)$

$x=2/3 \quad x=2$

| | $x<2/3$ | $2/3<x<2$ | $x>2$ |
|---|---|---|---|
| | + | − | + |
| 12 | + | + | + |
| $(3x-2)$ | − | + | + |
| $(x-2)$ | − | − | + |

$(2/3, 2)$

7. If $f''(x) = e^x(x+1)(x-2)^2$ , then the graph of f has inflection points when x=

a. -1 only
b. 2 only
c. -1 and 0 only
d. -1 and 2 only
e. -1, 0, 2 only

POI when $f''(x)$ changes sign - given $f''(x)$
$e^x$: always +; no POI
$(x-2)^2$: always +: no POI
$(x+1)$: changes sign
$x=-1$

8. The graph of y=f '' (x) on the interval (2,7) is shown above. ~~How many~~ points of inflections does f(x) have on this interval?

(a) One (b) Two (c) Three (d) Four (e) Five

9. A ball is thrown into the air along a path described by the equation $y = -\frac{x^2}{80} + x$ ,wherey is its height in feet above the ground. The maximum height that the ball reaches is

a. 20 ft
b. 30 ft.
c. 40ft
d. 60ft
e. 80ft

Max where deriv: + → -
EP and CP

$y' = \frac{-x}{40} + 1$

$0 = \frac{-x}{40} + 1$

$-1 = \frac{-x}{40}$

$40 = x$

plug back into y.

$y = \frac{-(40)^2}{80} + 40$

$y = 20$

10. Find two positive numbers whose product is a maximum if the sum of the numbers is 10.

a. 2 and 8
b. 3 and 7
c. 1 and 9
d. both are 5
e. none of these

$x + y = 10$ $y = 10 - x$
$P = xy$
$P = x(10-x)$
$P = 10x - x^2$
$P' = 10 - 2x$
CP: $x = 5$ $y = 5$

11. Given that $f(-3) = 2$ and $f'(-3)=-4$ , which of the following is the tangent line approximation of $f(-3.1)$?

Old Question!

a. 1.8
b. 2.2
c. 2.4
d. 2.8
e. 3

$y - y_1 = m(x - x_1)$

$y - (2) = -4(x+3)$

$y = -4(x+3)+2$

$y = -4(-3.1+3)+2$

$y = -4(-0.1)+2$

$y = 0.4+2 = 2.4$

16. Let $f(x)$ be a differentiable function for all x with $f(1) = -3$ and $f(5) = 4$. Which of the following must be true?

passes 0

f, not f'(x): so yes IVT

$(1,-3)$

$(5,4)$

$\frac{4-(-3)}{5-1} = \frac{7}{4}$

✓ I $f(k) = 0$ for some $k$ in (1, 5)

~~II~~ $f'(x) = \frac{7}{4}$ for all $x$ in (1, 5)

✓ III $f'(k) = \frac{7}{4}$ for some $k$ in (1, 5)

(a) I only (b) I and II (c) I and III (d) II and III (e) I, II, and III

17. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval (0,2), then c=

a. 0
b. ½
c. 1
d. 4/3
e. 2

$(0,0)$

$(2,0)$

$\frac{0-0}{2-0} = \frac{0}{2} = 0$

$f'(x) = 3x^2 - 4x$

$3x^2 - 4x = 0$

$x(3x-4) = 0$

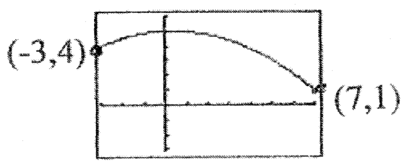
$x = 0 \quad x = \frac{4}{3}$

Calculator:

18. The graph of y = F(x) on the closed interval $\left[-3, 7\right]$ is shown below. F is continuous on the closed interval $\left[-3, 7\right]$ and differentiable on the open interval (-3,7). There exists a number "c" between -3 and 7 such that

MVT:

$$\frac{1-4}{7-(-3)} = \frac{-3}{10}$$




(a) $F'(c) = 0$ (b) $F'(c) = \frac{-3}{10}$ (c) $F'(c) = \cancel{\frac{3}{10}}$ (d) $F'(c) = \cancel{\frac{10}{3}}$ (e) $F'(c) = \cancel{\frac{-10}{3}}$

19. Two positive numbers have a sum of 60. What is the maximum product of one number times the square of the second number?

a. 3481
b. 3600
c. 27,000
d. 32,000
e. 36,000

$x + y = 60$ $x = 60 - y$

$xy^2 = P$

$(60-y)y^2 = P$

$60y^2 - y^3 = P$

$120y - 3y^2 = P'$

$\rightarrow$ CP: $120y - 3y^2 = 0$

$y(120 - 3y) = 0$

$y = 0$ or $y = 40$

$x = 60$ $x = 20$

nope b/c $xy^2 = 0$ max

21. The graph of the function $y = \frac{1}{3}x^3 - x^2 - 5x + 3\sin x$ has a point of inflection at

3.29 b. 2.21 c. 1.34 d. 0.41 e. -0.39

$y' = x^2 - 2x - 5 + 3\cos(x)$

$y'' = 2x - 2 - 3\sin(x)$

graph on calc:

find zero!

Bonus:

I. If g is a differentiable function such that g(x)<0 for all real numbers and and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

~~(a)~~ F has a relative maximum at x=-2 and a relative minimum at x=2

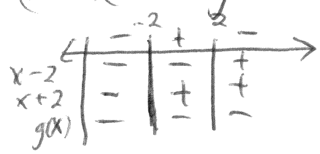
(b) F has a relative minimum at x=-2 and a relative maximum at x=2 ✓

~~(c)~~ F has relative maxima at x=-2 and x=2

~~(d)~~ F has a relative minima at x=-2 and x=2

~~(e)~~It cannot be determined if f has any relative extrema

$f'(x) = (x-2)(x+2)g(x)$ ← we know g(x) is −




slowing down

increasing

II. For all x in the closed interval [2,5] the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

| X | f(x) |
|---|---|
| 2 | 7 |
| 3 | 9 |
| 4 | 12 |
| 5 | 16 |

2, 3, 4 CCU

| X | f(x) |
|---|---|
| 2 | 16 |
| 3 | 13 |
| 4 | 10 |
| 5 | 7 |

Dec

| X | f(x) |
|---|---|
| 2 | 16 |
| 3 | 12 |
| 4 | 9 |
| 5 | 7 |

Dec

| X | f(x) |
|---|---|
| 2 | 16 |
| 3 | 14 |
| 4 | 11 |
| 5 | 7 |

Dec

| X | f(x) |
|---|---|
| 2 | 7 |
| 3 | 11 |
| 4 | 14 |
| 5 | 16 |

4, 3, 2 Inc CCD