Name______________________________ Date_______________________

AP Calculus AB Markwalter Work hard. Be nice.

Unit 4 Quiz/Test: Review Problems

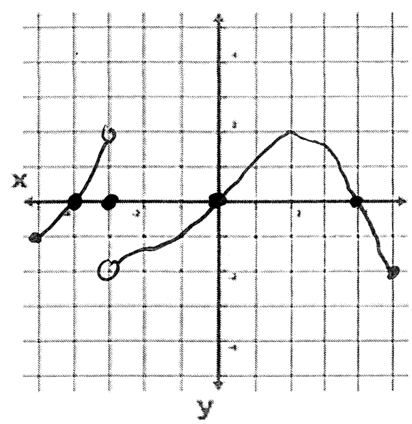
** means Calc OK

A. *Identify Critical Values*

KEY PROBLEM: Identify all the critical values for $f'(x) = (x-4)^2(3-x)e^x$ no CP

CP: $f'(x)=0$ or DNE

CP at $x = 4, 3$



1. How many critical values does $f(x) = (x-3)^3(x+1)^4$ have? Identify them.
   $f'(x) = (x-3)^3\,4(x+1)^3 + 3(x-3)^2(x+1)^4$
   $f'(x) = (x-3)^2(x+1)^3[4(x-3)+3(x+1)]$
   $f'(x) = (x-3)^2(x+1)^3(4x-12+3x+3)$
   $f'(x) = (x-3)^2(x+1)^3(7x-9)$ CP: $x=3$, $x=-1$, $x=\frac{9}{7}$

2. Identify all critical values of $f(x) = \frac{2}{3}x - x^{2/3}$.
   $f'(x) = \frac{2}{3} - \frac{2}{3}(x)^{-1/3}$
   $f'(x) = \frac{2}{3}[1 - x^{-1/3}]$ CP: when $x=0$ or $x=1$ > make $1-\frac{1}{\sqrt[3]{x}}=0$ or DNE
   $f'(x) = \frac{2}{3}\left(1 - \frac{1}{\sqrt[3]{x}}\right)$

3. Given the picture of f ' at right, identify all critical values of f(x).
   $f'(x)=0$ or DNE
   at $x = -3, -4, 0, 4$

B. *Identify extreme values on closed intervals*

KEY PROBLEM: The absolute minimum value of $F(x) = x^3 - 6x^2 - 1$ on the closed interval $[-1, 5]$ occurs at x=

this is a y-value (output)

(a) -1 (b) 0 (c) 2 $\boxed{\text{(d) } 4}$ (e) 5

Abs extrema are at CPs or EPs

$f'(x) = 3x^2 - 12x$

$f'(x) = 3x(x-4)$

CP: at $x = 0, 4$

$F(-1) = -8$

$F(0) = -26$

$F(4) = -1$

$F(5) = -33$ abs min

do first deriv sign chart

KEY PROBLEM: Which of the following descriptions is true about the extrema of $f(x) = x \cdot e^x$?

a. f(x) has a local max at x= -1
b. f(x) has a local min at x= -1
c. f(x) has a local max ~~at x=1~~
d. f(x) has a local min ~~at x=1~~
e. f(x) has ~~no local extr~~ema

$f'(x) = xe^x + e^x = e^x(x+1)$

CP: $x = -1$

	$\ominus$ -1	$\oplus$
e^x	+	+
$(x+1)$	−	+

local min at $x=-1$

$f'(x)$ changes $- \to +$

y-value

4. Identify the absolute minimum value of $f(x) = x^4 - 4x^3$ on [-1,4] and where it occurs.

EPs: $x=-1$, $x=4$

$f'(x) = 4x^3 - 12x^2$

$f'(x) = 4x^2(x-3)$

CP: $x=0$, 3

CP: $x=0$, $x=3$

$f(-1)=5$
$f(0)=0$
$f(3)=-27$
$f(4)=0$

The absolute min is $\boxed{-27}$

5. The absolute maximum value of $f(x) = \frac{16}{x} + x^2$ on [1,4] is:

EP: $x=1$, $x=4$

CP: $x=2,-2$, $x=0$

$f(x) = 16x^{-1} + x^2$

$f'(x) = -16x^{-2} + 2x$

$f'(x) = x^{-2}[-16 + 2x^3]$

$f'(x) = \frac{1}{x^2}[2x^3 - 16]$

$2x^3 - 16 = 0$

$2x^3 = 16$

$x = \sqrt[3]{8}$

CP: $x=2$, $x=0$

$f(0) \Rightarrow$ Not in interval
$f(1)=17$
$f(2)=12$
$f(4)=20$

Max is $\boxed{20}$

C. Identify intervals where functions are increasing, decreasing, or constant.

KEY PROBLEM: For which interval(s) is the function $F(x) = x^3 - 3x^2 - 9x + 2$ increasing? $f'(x)$ is +

(a) (-1,3) (b) $(-\infty,-1)$ (c) $(-\infty,-1),(3,\infty)$ (d) $(-\infty,3)$ (e) $(3,\infty)$

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 3(x^2 - 2x - 3)$

$f'(x) = 3(x-3)(x+1)$

CP: $x=3,-1$

	+	-1 −	3 +
$(x-3)$	−	−	+
$(x+1)$	−	+	+
	↗	↘	↗

inc: $(-\infty,-1)$, $(3,\infty)$

6. Identify where $f(x) = x^4 - 4x^3$ is increasing. Identify where it is decreasing.

$f'(x) = 4x^3 - 12x^2$

$f'(x) = 4x^2(x-3)$

CP: $x=0$, $x=3$

	$\ominus$	0 $\ominus$	3 $\oplus$
$4x^2$	+	+	+
$(x-3)$	−	−	+
	↘	↘	↗

inc: $(3,\infty)$ b/c $f'(x)$ +

dec: $(-\infty,0)$ $(0,3)$ b/c $f'(x)$ −

$f(x)=x^3+48x^{-1}$

7. Identify where $f(x) = x^3 + \frac{48}{x}$ is increasing. Identify where it is decreasing.

$f'(x)=3x^2-48x^{-2}$

$f'(x)=3x^{-2}(x^4-16)$

$f'(x)=\frac{3}{x^2}(x^4-16)$

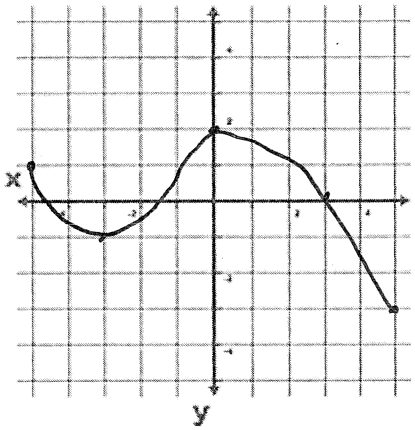
CP: $x=0$

$x=\pm 2$

	$+$ -2	$-$ 0	$-$ 2	$+$
$\frac{3}{x^2}$	$+$	$+$	$+$	$+$
(x^4-16)	$+$	$-$	$-$	$+$

inc: $(-\infty,-2)(2,\infty)$ Dec: $(-2,0)(0,2)$

$f'(x)$



8. Given the picture of f '(x) at right, determine where f (x) is increasing and decreasing. Explain how you know based on the graph of f '(x).

f(x) Inc when $f'(x)$ is $+$: $(-5,-4.5)$, $(-1.5,3)$

f(x) dec when $f'(x)$ is $-$: $(-4.5,-1.5)$ $(3,5)$

D. *Identify local (relative) maximum/minimum values (using 1st or 2nd Derivative Test as you please…☺).*

$f'(x): - \longrightarrow +$ first derivative test.

KEY PROBLEM: At what value(s) of x does $F(x) = x^4 - 8x^2$ have a relative minimum?

(a) -2 and 0 (b) -2 and 2 (c) 0 (d) -2 , 0 , and 2 (e) 0 and 2

$f'(x)=4x^3-16x$

$f'(x)=4x(x^2-4)$

$f'(x)=4x(x-2)(x+2)$

$x=0,2,-2$

	$-$ -2	$+$ 0	$-$ 2	$+$
$4x$	$-$	$-$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	$+$
$(x+2)$	$-$	$+$	$+$	$+$
	↘	↗	↘	↗

KEY PROBLEM: Let $f(x)$ be a polynomial function such that $f(-2) = 6$, $f'(-2)= 0$, and $f''(-2) = -4$.

CP CCD

The point (-2, 6) is a(n)

a) local maximum b) local minimum c) $y-$intercept d) point of inflection e) $x-$intercept

↳ this is the second derivative test.

9. Identify the local minimum and maximum values of $f(x) = 2x^4 - \frac{4}{3}x^6$. Also, identify the x-values where they occur. we want both (x,y)

$f'(x)=8x^3-8x^5$

$f'(x)=8x^3(1-x^2)$

$f'(x)=8x^3(1-x)(1+x)$

CP: $x=0,1,-1$

	$+$ -1	$-$ 0	$+$ 1	$-$
$8x^3$	$-$	$-$	$+$	$+$
$(1-x)$	$+$	$+$	$+$	$-$
$(1+x)$	$-$	$+$	$+$	$+$
	↗	↘	↗	↘

Rel Maxima at: Max is:

$x=-1$ $f(-1)=2-\frac{4}{3}$

$x=1$ $f(1)=2-4/3$

Rel Min at Min is:

$x=0$ $f(0)=0$

10. Suppose $f'(x) = x^2(x+3)^3e^x$ Identify where f(x) has any local extreme values given f(x) is continuous.

$f'(x) = x^2(x+3)^3e^x$

CP: $x=0$

$x=-3$

	$-$	-3	$+$	0	$+$
x^2	+		+		+
$(x+3)^3$	−		+		+
e^x	+		+		+

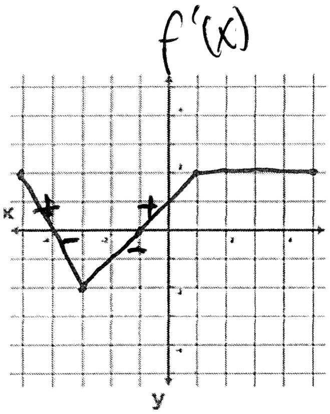
Local min at $x=-3$
b/c $f'(x)$ $- \to +$

No local max

11. Suppose f(x) is a polynomial function such that f(4)=8, f'(4)=0 and f''(4)=4. What do we know occurs at x=4 and why?

CP CCU

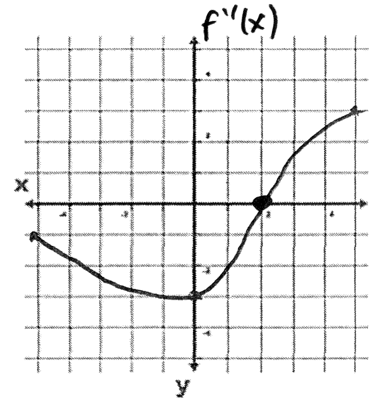
A relative minimum of f(x) occurs at $x=4$ b/c there is a critical point and the graph of f is CCU at that x-value.



12. Given the picture of f '(x) at left, determine the x-values where the local extrema of f(x) occur on (-5,5).Identify whether they are a max or a min and explain how you know based on f '(x).

local min occurs at $x=-1$ b/c $f'(x)$ changes from negative to positive.

local max occurs at $x=-4$ b/c $f'(x)$ changes from positive to negative.



CP

13. Given the picture of f ''(x) at right and f '(2)=0, determine whether f(x) has a max or min at x=2.

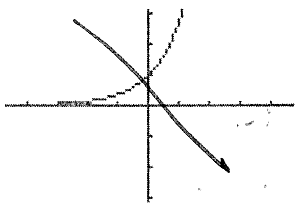
f(x) has neither a min nor a max at $x=2$ b/c $f''(x)$ is 0 (f is neither CCU nor CCD)

E. *Match f with f ' and f '' or sketch a graph of f(x) given f '(x) and f ''(x).*

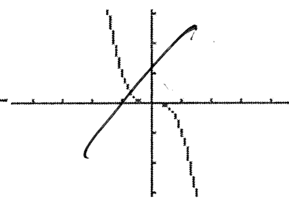
KEY PROBLEM: If $f'(x) > 0$ and $f''(x) < 0$ for x<0 and $f''(x) > 0\ for\ x > 0$, which of the following could be the graph of f ?

f inc f CCD f CCU

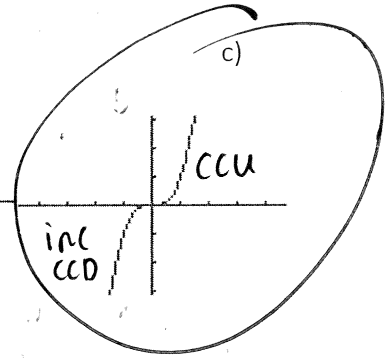
a)



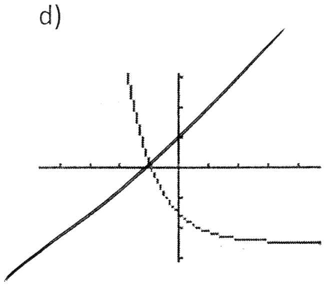
b)



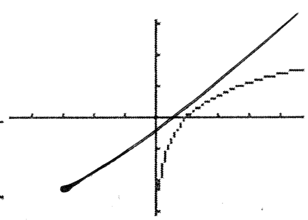
c)



d)



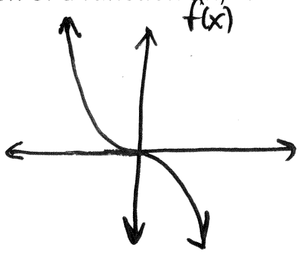
e)



f is dec f CCU

14. Sketch the graph of a function f(x) for which f'(x)<0 for all x and f''(x)>0 for x:(-∞,0) and f''(x)<0 for x:(0,∞).

f CCD

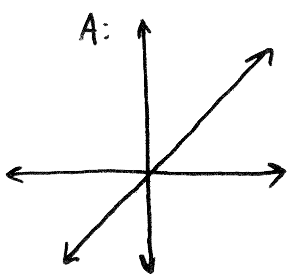


15. A. Sketch the graph of a function that is increasing at a constant rate for all x.

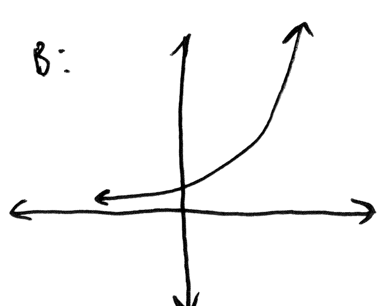
B. Sketch the graph of a function that is increasing (f'(x)) at an increasing rate (f''(x)>0) for all x.

C. Sketch the graph of a function that is increasing (f'(x)) at a decreasing rate (f''(x)<0) for all x.

A:

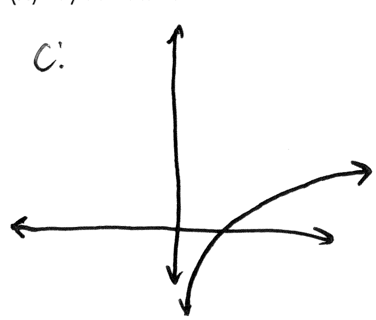


B:



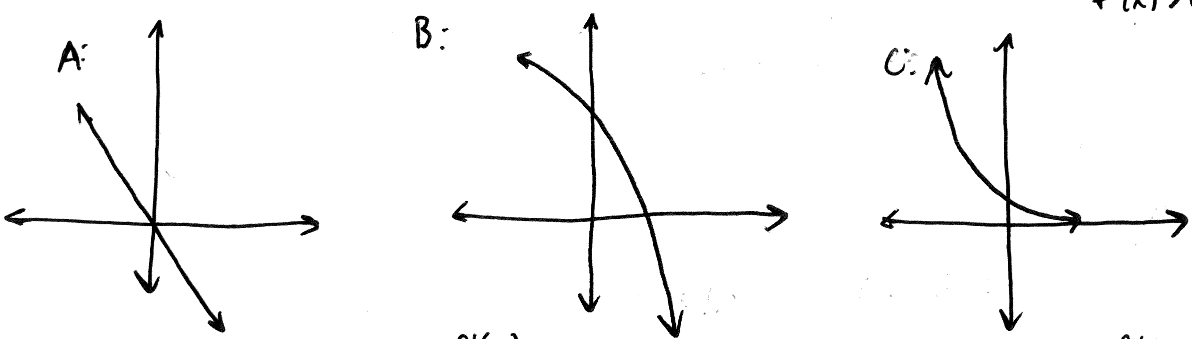
f'(x)>0 and f''(x)>0

C:



f'(x)>0 and f''(x)<0

16. A. Sketch the graph of a function that is decreasing at a constant rate for all x. change $f''(x)<0$
B. Sketch the graph of a function that is decreasing ($f'(x)<0$) at an increasing rate ~~($f''(x)>0$)~~ for all x.
C. Sketch the graph of a function that is decreasing ($f'(x)<0$) at a decreasing rate ($\cancel{f''(x)<0}$) for all x. $f''(x)>0$



$f'(x)<0$ and $f''(x)<0$ $\qquad$ $f'(x)<0$ and $f''(x)>0$

F. ***Identify where a function f(x) is concave up/down and has points of inflection***

KEY PROBLEM: Which of the following are all the intervals on which the graph of $f(x)=\dfrac{x-1}{x+3}$ is concave upward? $f''(x)>0$

a) $(-\infty,\infty)$ b) $(-\infty,-3)$ c) $(1,\infty)$ d) $(-3,\infty)$ e) $(-\infty,1)$

$$f'(x)=\frac{(x+3)(1)-(x-1)(1)}{(x+3)^2}$$

$$f'(x)=\frac{x+3-x+1}{(x+3)^2}=\frac{4}{(x+3)^2}$$

$$f''(x)=\frac{(x+3)^2(0)-4\cdot 2(x+3)}{(x+3)^4}$$

$$f''(x)=\frac{-8(x+3)}{(x+3)^4}$$

$$f''(x)=\frac{-8}{(x+3)^3}$$

	⊕	-3 ⊖
-8	−	−
$(x+3)^3$	−	+

$(-\infty,-3)$

$x=-3$

KEY PROBLEM:

How many points of inflection does the graph of $y=x^6-10x^4+2x-1$ have? $f''(x)$ changes sign

a) 0 b) 1 c) 2 d) 3 e) 4

$y'=6x^5-40x^3+2$

$y''=30x^4-120x^2$

$y''=30x^2(x^2-4)$

$y''=30x^2(x-2)(x+2)$

$x=0 \quad x=2 \quad x=-2$

	+	-2 −	0 −	2 +
$30x^2$	+	+	+	+
$(x-2)$	−	−	−	+
$(x+2)$	−	+	+	+

POI at $x=-2,2$
two POIs

Shortcut! $x=0$ will NOT be a POI b/c its factor is x^2 [always positive]. So $(x-2)$ and $(x+2)$ only give us POI.

17. On which intervals is the graph of $f(x) = \frac{x+2}{x-1}$ concave up and concave down?

$f'(x) = \frac{(x-1)-(x+2)}{(x-1)^2} = \frac{x-1-x-2}{(x-1)^2} = \frac{-3}{(x-1)^2}$

$f''(x) = \frac{(x-1)^2(0)-(-3)(2(x-1))}{(x-1)^4} = \frac{3[2(x-1)]}{(x-1)^4} = \frac{6}{(x-1)^3}$ $x=1$

	⊖	⊕
6	$+$	$+$
$(x-1)^3$	$-$	$+$

CCD: $(-\infty, 1)$ $f''(x)<0$

CCU: $(1, \infty)$ $f''(x)>0$

18. Identify the points of inflection of $f(x) = x^{1/3} + 2$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f''(x) = -\frac{2}{9}x^{-5/3} = \frac{-2}{9\sqrt[3]{x^5}} \Rightarrow x=0$

	⊕	⊖
-2	$-$	$-$
$9\sqrt[3]{x^5}$	$-$	$+$

POI at $x=0$

19. The graph of $y = x + \frac{1}{x}$ is both decreasing and concave up on what interval?

$y = x + x^{-1}$ $f'(x)<0$ $f''(x)>0$

$y' = 1 - x^{-2}$

$y' = 1 - \frac{1}{x^2}$

CP: $x=0$, $x=1$, $x=-1$

$\left(1-\frac{1}{x^2}\right)$: $+$ -1 $-$ 0 $-$ 1 $+$

Dec: $(-1,0)$ $(0,1)$

$y'' = 2x^{-3}$

$y'' = \frac{2}{x^3}$

	$-$	$+$
2	$+$	$+$
x^3	$-$	$+$

CCU: $(0, \infty)$

$\boxed{\text{Both: } (0,1)}$

20. **On what interval(s) is the graph of $f(x) = 3x^{\frac{2}{3}} - 4x + 7$ concave down?

graph

$f'(x) = 2x^{-1/3} - 4$

$f''(x) = -\frac{2}{3}x^{-4/3}$

Graph: Look for where below x-axis OR

	⊖	⊖
$-\frac{2}{3}$	$-$	$-$
$\frac{1}{\sqrt[3]{x^4}}$	$+$	$+$

CCD: $(-\infty, 0)$ $(0, \infty)$

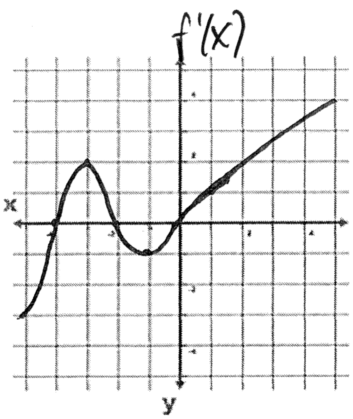
21. ** The second derivative of function f is given by $f''(x) = e^{-x} - \sin x$. How many points of inflection does f have on the interval $0 \le x \le 10$?

Graph on the interval $(0, 10)$

Count how many times

$f''(x)$ changes sign (crosses x-axis)

Four points of inflection.



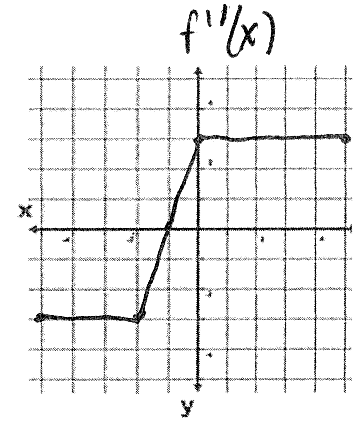
25. Given the graph of f'(x) at left, identify where f(x) is concave up, Concave down, and has points Of inflection.

CCU when $f'(x)$ is increasing $\therefore (-5,-3)$ $(-1,5)$

CCD when $f'(x)$ is decreasing $\therefore (-3,-1)$

POI when $f'(x)$ has maxima/minima or $f'(x)$'s slope changes sign.

$\therefore$ at $x = -3, -1$



26. Given the graph of f''(x) at right, identify where f(x) is concave up, concave down And has points of inflection.

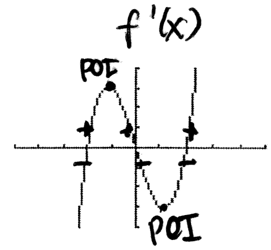
CCU when $f''(x)$ is $+$ $\therefore (-1, 5)$

CCD when $f''(x)$ is $-$ $\therefore (-5,-1)$

POI when $f''(x)$ changes sign $\therefore$ at $x=-1$

EXTRA PROBLEMS:

Questions 14 -16 refer to the graph of $f'(x)$ shown at right:



when $f'(x)$ - → +

14. For what value(s) of x does f(x) achieve a relative minimum?

a) x=1 b) x=0 c) x=-2 only d) x=2 only e) x= -2 and x=2

$f'(x)=0$

15. For what value(x) of x does f(x) have horizontal tangents?

a) X= -2, x=0, and x=2 b) x= -1 and x=1 c) x= -2,-1,0,1, and 2

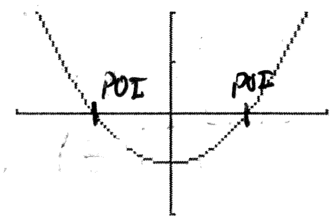
d) x=0 only e) x= -2 and 2 only

16. How many points of inflection does the graph of f(x) have? $f'(x)$ has extrema

a) 0 b) 1 c) 2 d) 3 e) Not enough info.

Questions 17 -19 refer to the graph of $y = f''(x)$ shown at right:

$f''(x)$



17. On which intervals is the graph of f concave up? $f''(x)>0$

a) $(-\infty,0)$ b) $(0,\infty)$ c) $(-1,1)$ d) $(-\infty,-1),(1,\infty)$ e) $(-\infty,\infty)$

→ when $f''(x)$ is negative

18. On which intervals is f' decreasing?

a) $(-\infty,0)$ b) $(0,\infty)$ c) $(-1,1)$ d) $(-\infty,1),(1,\infty)$ e) $(-\infty,\infty)$

$f''(x)$ changes sign

19. How many points of inflection does f(x) have?

a) 0 b) 1 c) 2 d) 3 e) not enough info

20. If $f''(x) = (x-1)(x+2)^3(x-4)^2$, then the graph of f has inflection points when x=

$x=1$ ✓ $x=-2$ ✓ doesn't change sign

a) -2 only b) 1 only c) 1 and 4 only d) -2 and 1 only e) -2, 1, and 4 only

or do sign chart