Name: Answers

Unit 6: Practice Problems 2

Part A: No Calculators

The table shows some values of continuous function f and its first derivative. Evaluate $\int_8^0 f'(x)\,dx$.

x	$f(x)$	$f'(x)$
0	11	3
2	15	2
4	16	–1
6	12	–3
8	7	0

(A) –1/2 **(B)** –3/8 **(C)** 3
(D) 4 **(E)** none of these

$$\int_8^0 f'(x)dx = f(x)\Big|_8^0 = f(0)-f(8) = 11-7=4$$

2.

Reminder: $$\int_a^b f(x)dx = \lim_{x\to\infty}\sum_{i=1}^{n} f(a+\Delta x\cdot i)\Delta x \text{ , where } \Delta x = \frac{b-a}{n}$$

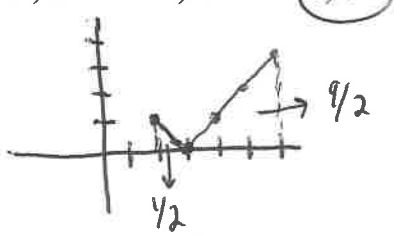
Write the left Riemann Sum approximation of $\int_1^7 (4\sin(x)+2)\,dx$ if the approximation has n subintervals of equal length.

$\Delta x = \frac{7-1}{n}$

$$\lim_{x\to\infty}\sum_{k=1}^{n} f\left(1+\frac{6k}{n}\right)\frac{6}{n} = \lim_{x\to\infty}\sum_{k=1}^{n}\left(4\sin\left(1+\frac{6k}{n}\right)+2\right)\frac{6}{n}$$

3. $\int_2^6 |x-3|\,dx =$

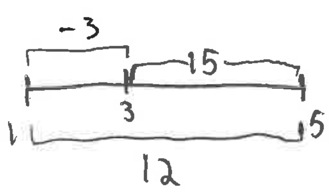
A) 4 B) 2 C) 3 D) 5 E) 6



$\frac{1}{2}+\frac{9}{2}=5$

4. If $\int_1^5 F(x)dx = 12$ and $\int_1^3 F(x)dx = -3$, then $\int_3^5 F(x)dx =$

A) - 12 B) - 9 C) 9 D) -15 E) 15



5. $\int (x^3 - 4x)dx =$ $\frac{1}{4}x^4 - 2x^2 + C$

a. $3x^2 - 4 + C$

b. $4x^4 - 8x^2 + C$

c. $\frac{x^4}{3} - 4x^2 + C$

d. $\frac{x^4}{4} - 4x + C$

e. $\frac{x^4}{4} - 2x^2 + C$

Fast u-sub!

6. If $\frac{dy}{dx} = \sin(2x)$, then y= $-\frac{1}{2}\cos(2x) + C$

a. $\frac{1}{2}\cos(2x) + C$

b. $\cos(2x) + C$

c. $-\frac{1}{2}\cos(2x) + C$

d. $\frac{1}{2}\sin^2(2x) + C$

e. $-\frac{1}{2}\sin^2(2x) + C$

u-sub

7. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3x^2}\, du = dx$

$\int x^2 \cos(u) \frac{1}{3x^2}\, du = \int \frac{1}{3}\cos(u)\, du = \frac{1}{3}\sin(u) + C$

$= \frac{1}{3}\sin(x^3) + C$

u-sub

8. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3x^2}du = dx$

$\int \frac{3x^2}{\sqrt{u}} \frac{1}{3x^2} du =$

$\int u^{-1/2} du = 2u^{1/2} + C$

$= 2\sqrt{x^3+1} + C$

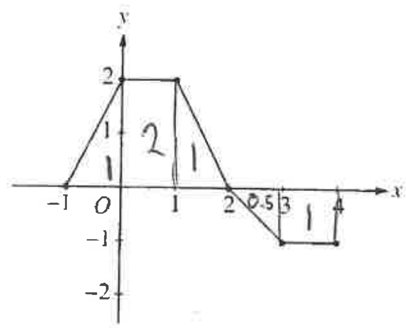
a rule in 6.7 packet

9. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}dx$?

a. $\arcsin\frac{x}{5} + C$ b. $\arcsin x + C$ c. $\frac{1}{5}\arcsin\frac{x}{5} + C$ d. $\sqrt{25-x^2} + C$ e. $2\sqrt{25-x^2} + C$

$\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1}\left(\frac{u}{a}\right) + C$

10.



The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of

$\int_4^{-1} f(x)dx = -\int_{-1}^{4} f(x)dx = -(4-1.5) = -2.5$

a. 5.5 b. -5.5 c. 3.5 d. -2.5 e. 2.5

$\int_4^k x^2 dx = \frac{1}{3}x^3\Big|_4^k = \frac{1}{3}k^3 - \frac{64}{3} = 0$

$\frac{1}{3}k^3 = \frac{64}{3}$

$k^3 = 64$

$k = 4$

11. What are **all** values of k for which $\int_4^k x^2 dx = 0$?

A) -4 B) 0 C) 4 D) -4 and 4 E) -4, 0, and 4

12.. $\int_1^2 \frac{1}{x^3}dx = \int_1^2 x^{-3}dx = -\frac{1}{2}x^{-2}\Big|_1^2 = \left(-\frac{1}{2}\cdot 2^{-2}\right)-\left(-\frac{1}{2}\cdot 1^{-2}\right) = \left(\frac{-1}{8}\right)+\frac{1}{2} = \frac{3}{8}$

A) $-\frac{7}{8}$ B) $\frac{3}{8}$ C) $\frac{-3}{4}$ D) $\frac{-3}{8}$ E) ln7

13. Using the substitution u=$x^2 + 4$, $\int_0^2 \frac{x}{\sqrt{x^2+4}}dx$ is equivalent to:

a. $\frac{1}{2}\int_0^2 \frac{1}{\sqrt{u}}du$ b. $\int_0^2 \frac{1}{\sqrt{u}}du$ c. $\frac{1}{2}\int_4^8 \frac{1}{\sqrt{u}}du$ d. $2\int_4^8 \frac{1}{\sqrt{u}}du$ e. $2\int_0^2 \frac{1}{\sqrt{u}}du$

$u = x^2+4$ $\quad x=2 \to u=8$

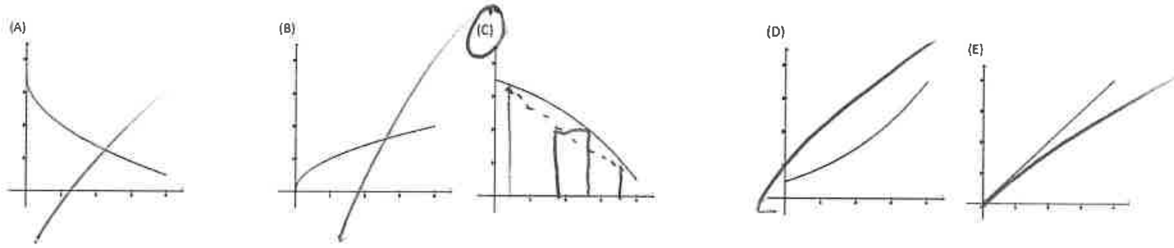
$\frac{du}{dx} = 2x$ $\quad x=0 \to u=4$

$\frac{1}{2x}du = dx$

$\int_4^8 \frac{x}{\sqrt{u}}\cdot\frac{1}{2x}du = \int_4^8 \frac{1}{2}\cdot\frac{1}{\sqrt{u}}du = \frac{1}{2}\int_4^8 \frac{1}{\sqrt{u}}du$

→ TRAP under approx if f(x) is CCD

14. If both a trapezoidal sum and Right Riemann Sum $\boxed{\text{under approximates}}$ $\int_0^4 f(x)dx$ which of the following could be the graph of y=f(x)? → if f(x) is dec



15. Find $\frac{d}{dx}[\int_0^{3x} \sin(2t+1)\,dt$ $= \sin(2(3x)+1)\cdot 3 = \sin(6x+1)\cdot 3$

a. $\sin(6x+1)$
b. $\sin(6x)$
c. $3\sin(6x+1)$
d. $\frac{1}{3}\sin(6x+1)$

16. Evaluate the following integrals

Partial Fractions $\int \frac{5x-3}{x^2-2x-3}\,dx = \int \frac{A}{(x-3)} + \frac{B}{(x+1)}\,dx = \int \frac{3}{(x-3)} + \frac{2}{(x+1)}\,dx = 3\ln|x-3| + 2\ln|x+1| + C$

$5x-3 = A(x+1) + B(x-3)$

$x=-1$ $\quad B=2$

$x=3$ $\quad A=3$

Int by Parts $\int x^2 \ln x\,dx$

$u = \ln(x)$ $\quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{1}{3}x^3$

$\int x^2\ln(x)\,dx = \ln(x)\cdot\frac{1}{3}x^3 - \int \frac{1}{3}x^3\cdot\frac{1}{x}\,dx$

$= \frac{x^3}{3}\ln(x) - \int \frac{1}{3}x^2\,dx$

$= \frac{x^3}{3}\ln(x) - \frac{1}{9}x^3 + C$

u-sub $\int x^3\sqrt{4+x^4}\,dx = \int x^3\sqrt{u}\,\frac{1}{4x^3}\,du = \int \frac{1}{4}\sqrt{u}\,du = \int \frac{1}{4}u^{1/2}\,du = \frac{1}{6}u^{3/2} + C$

$= \frac{1}{6}(4+x^4)^{3/2} + C$

$u = 4+x^4$

$\frac{du}{dx} = 4x^3$

$\frac{1}{4x^3}\,du = dx$

Improper $\int_{-\infty}^{\infty} \cos(\pi t)\,dt = \lim_{a\to-\infty}\int_a^0 \cos(\pi t)\,dt + \lim_{b\to\infty}\int_0^b \cos(\pi t)\,dt$

$= \lim_{a\to-\infty}\left(\frac{\sin(\pi t)}{\pi}\right)\Big|_a^0 + \lim_{b\to\infty}\left(\frac{\sin(\pi t)}{\pi}\right)\Big|_0^b$

$= \lim_{a\to-\infty}\left(0 - \frac{\sin(\pi a)}{\pi}\right) + \lim_{b\to\infty}\left(\frac{\sin(\pi b)}{\pi} - 0\right)$

Uh Oh! $\sin(\pi a)$ keeps oscillating, so does $\sin(\pi b)$; so diverges.

Improper $\int_1^{\infty} \frac{1}{(3x+1)^2}\,dx = \lim_{a\to\infty}\int_1^a \frac{1}{(3x+1)^2}\,dx = \lim_{a\to\infty}\int_4^{3a+1}\frac{1}{u^2}\,\frac{1}{3}\,du = \lim_{a\to\infty}\frac{1}{3}\int_4^{3a+1} u^{-2}\,du$

$u = 3x+1$ $\quad x=a \to u = 3a+1$

$\frac{du}{dx} = 3$ $\quad x=1 \to u=4$

$\frac{1}{3}\,du = dx$

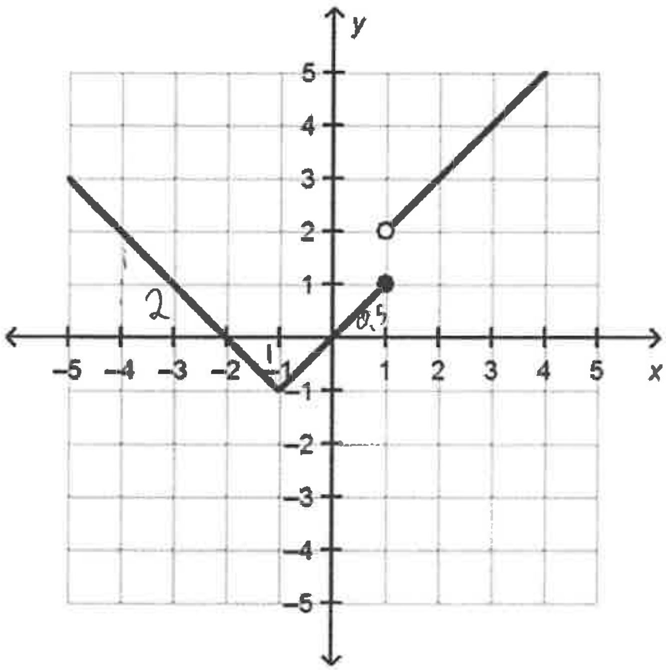
$= \lim_{a\to\infty}\frac{1}{3}\left(-u^{-1}\right)\Big|_4^{3a+1}$

$= \lim_{a\to\infty}\frac{1}{3}\left(\frac{-1}{3a+1}\right) - \frac{1}{3}\left(\frac{-1}{4}\right)$

0

$= \frac{1}{12}$

The graph of f(x) is given below. If $g(x) = \int_{-4}^{x} f(t)\,dt$, answer questions 4-6.



$g(x) = \int_{-4}^{x} f(t)dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

17. What is the value of g(1)?

$g(1) = \int_{-4}^{1} f(t)dt = 2 - 1 + 0.5$

Areas!

a. 1
b. 1.5
c. 2
d. 2.5

18. What is the value of g'(0)?

$g'(0) = f(0) = 0$

point on f

a. -1
b. 0
c. 1
d. 2

19. Which of the following is true?

a. The function g has a relative maximum at x=-2 yes b/c $g'(x) = f(x)$ changes $+ \to -$,
b. The function g has a relative maximum at x=0
c. The function g has a relative minimum at x=-2
d. The function g has a point of inflection at x=1

Calc Ok

20.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

a. 8 b.7 c. 9 d. 11 e) 12

$$\int_0^2 f(x)\,dx = \frac{(3+3)\frac{1}{2}}{2} + \frac{(3+5)\frac{1}{2}}{2} + \frac{(5+8)\frac{1}{2}}{2} + \frac{(8+13)\frac{1}{2}}{2} = \frac{3}{2}+2+\frac{13}{4}+\frac{21}{4}$$

$$= \frac{3}{2}+2+\frac{34}{4}$$

$$= \frac{3}{2}+2+\frac{17}{2} = 12$$

Trap: $\frac{(y_1+y_2)\Delta x}{2} + \ldots$

21. Use a left hand Riemann sum with 3 subintervals of equal width to approximate the value of $\int_0^6 f(x)\,dx$. Selected values of the function are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	1	2	5	10	17	26	37

A) 118 B) 76 C) 46 D) 38 E) 18

$$\int_0^6 f(x)\,dx = 2(1)+2(5)+2(17) = 2+10+34$$

$\Delta x = 2$

22.

Time (milliseconds)	0	5	10	15	20	25	30
Light output (millions of lumens)	0	.2	.6	2.6	4.2	3	1.8

The data in the table represents the rate of light output of a flash bulb at a given time. Using a midpoint approximation with 3 equal subdivisions estimate the total light output of the bulb, measured in million lumen-milliseconds.

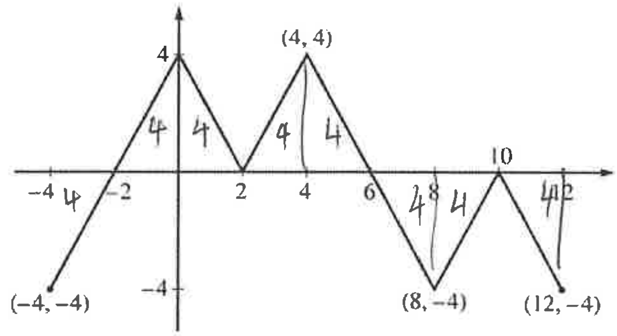
(A) 5.8 (B) 48 (C) 57 (D) 58 (E) 66

$\Delta x = 10$

$$10(0.2)+10(2.6)+10(3) = 2+26+30 = 58$$

Non-Calculator Free Response

g(x) is area
g'(x) is point
g''(x) is slope



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

a) $g'(x) = f(x)$, so g has neither a relative min nor a relative max at $x=10$ b/c $g'(x) = f(x)$ does not change sign.

b) g has a point of inflection at $x=4$ b/c $g''(x) = f'(x)$ changes sign at $x=4$ [could say $g'(x) = f(x)$ changes from increasing to decreasing].

c) Absolute Extrema

EndPoints $x=-4$
$x=12$

Critical Points
$g'(x) = f(x) = 0$ or DNE
$x=-2$
$x=2 \rightarrow$ not a min nor max
$x=6$
$x=10 \rightarrow$ not a min nor max

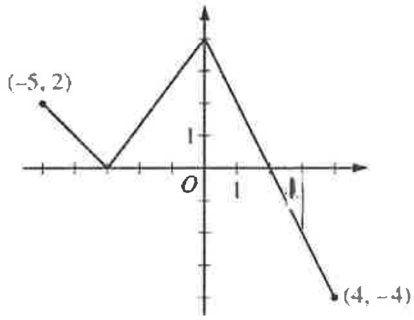
$g(-4) = \int_2^{-4} f(t)\,dt = -4$
$g(-2) = \int_2^{-2} f(t)\,dt = -8$
$g(6) = \int_2^{6} f(t)\,dt = 8$
$g(12) = \int_2^{12} f(t)\,dt = -4$

Abs Min is $g(-2) = -8$. Abs max is $g(6) = 8$

d) $g(2) = 0$ and $g(10) = 0$

so $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$

$\downarrow$
integral value



Graph of f

$g(x)$ is area
$g'(x)$ is point
$g''(x)$ is slope

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$

b) g is increasing when $g'(x) = f(x)$ is positive. g is concave down when $g''(x) = f'(x)$ is negative. This happens on $(-5,-3)$ and $(0,2)$ b/c $g'(x) = f(x)$ is positive and decreasing.

c) $h(x) = \frac{g(x)}{5x}$ $\quad h'(x) = \frac{5x\, g'(x) - g(x)\cdot 5}{25x^2} \Rightarrow h'(3) = \frac{15g'(3) - g(3)\cdot 5}{25(9)}$

quotient rule

$g'(3) = -2$
$g(3) = 9$

$$\boxed{h'(3) = \frac{15\cdot(-2) - 9\cdot 5}{25(9)}}$$

d) $p(x) = f(x^2 - x)$

chain rule

$p'(x) = f'(x^2 - x)(2x - 1)$
$p'(-1) = f'(2)(-3) = -2(-3) = \boxed{6}$

↳slope of f at $x = 2$