Winter Break MC Answers

(1.) $\boxed{E}$ H.A. Check $\lim_{x\to\infty} \frac{\boxed{2x}}{1x-3} = \frac{2x}{1x} = 2$ $\quad \lim_{x\to-\infty} \frac{2x}{1x-3} = \frac{2x}{1x} = 2$

The graph of $y = \frac{2x}{x+3}$ gets closer to $y=2$ as $x\to\infty$ or $x\to-\infty$

(2.) $\boxed{A}$ $\lim_{x\to 2} \frac{10x-20}{x^2-7x+10} = \lim_{x\to 2} \frac{10\cancel{(x-2)}}{\cancel{(x-2)}(x-5)} = \lim_{x\to 2} \frac{10}{x-5} = \frac{10}{-3}$

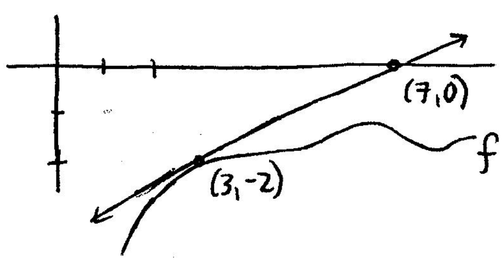
(3.) $\boxed{C}$ $f(x) = (9+x^2)^5$ $\quad f'(x) = 5(9+x^2)^4(2x) = 10x(9+x^2)^4$

(4.) $\boxed{B}$ f increasing for $2<x<6$ since $f'(x)>0$ on $2<x<6$
(positive)

(5.) $\boxed{D}$ $y = \frac{x^2}{2x+1}$ $\quad y' = \frac{(2x+1)(2x)-(x^2)(2)}{(2x+1)^2}$ Quotient rule.

$y' = \frac{4x^2+2x-2x^2}{(2x+1)^2} = \frac{2x^2+2x}{(2x+1)^2} = \frac{2x(x+1)}{(2x+1)^2}$

(6.) $\boxed{D}$



$m = \frac{\Delta y}{\Delta x} = \frac{0 - -2}{7-3} = \frac{2}{4} = \frac{1}{2}$

(7.) $\boxed{B}$ "slope of tangent to the curve = value of y'"

$y = \ln(e^{x^2}+2x)$

$y' = \frac{1}{e^{x^2}+2x}(e^{x^2}\cdot 2x+2)$ at y-intercept $x=0$

$y' = \frac{1}{e^0+2(0)}(e^0\cdot 2\cdot 0+2) = \frac{2}{1}$

(8.) $\boxed{A}$ $\int x\sec^2(3x^2)dx = \int \sec^2(3x^2)\cdot x\,dx$

$u = 3x^2$

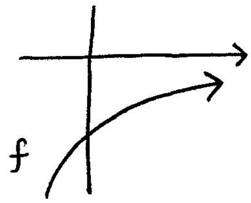
$du = 6x\,dx$

$\frac{1}{6}du = x\,dx$

$\Rightarrow \frac{1}{6}\int \sec^2(u)\,du = \frac{1}{6}\tan(u) + C$

$\boxed{= \frac{1}{6}\tan(3x^2) + C}$

9. $\boxed{C}$



f is below x axis so $f<0$ ✓
f is increasing so $f'>0$
f is concave down so $f''<0$ ✓

10. $\boxed{D}$

*product rule!

$\frac{d}{dt}\left(e^{2t}\cdot\cos(3t)\right) = e^{2t}\cdot(-\sin(3t)\cdot 3) + \cos(3t)e^{2t}\cdot 2$

*factor out e^{2t} $\quad \frac{dy}{dt} = e^{2t}\left[-3\sin(3t) + 2\cos(3t)\right]$

11. $\boxed{A}$ Tangent line equation: $y-y_1=m(x-x_1)$ $\quad m=f'$

$f(x) = 4-12\sqrt{x} = 4-12x^{1/2}$
$f(4) = 4-12\sqrt{4}$
$f(4) = 4-12(2) = 4-24 = -20$

$f'(x) = 0-12(\frac{1}{2}x^{-1/2})$
$f'(x) = \left.\frac{-6}{\sqrt{x}}\right|_{x=4} = \frac{-6}{\sqrt{4}} = \frac{-6}{2} = -3$

$y - -20 = -3(x-4) \Rightarrow y+20 = -3(x-4) \Rightarrow y = -3x+12-20$

12. $\boxed{C}$ $f(x) = \begin{cases} 4x-3, & x<3 \\ x^2, & x\geq 3 \end{cases}$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} 4x-3 = 9$
$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x^2 = 9$
f continuous! at $x=3$.

$\lim_{x\to 3^-} f'(x) = \lim_{x\to 3^-} 4 = 4$
$\lim_{x\to 3^+} f'(x) = \lim_{x\to 3^+} 2x = 6$
f not differentiable at $x=3$ since right + left hand slope at 3 not equal!

13. $\boxed{E}$ $x(t) = 2-t^3+\frac{1}{2}t^4$ (position) at rest means velocity $=0$.

$v(t) = x'(t) = -3t^2+\frac{1}{2}(4)t^3 = -3t^2+2t^3 = t^2(-3+2t) = 0$ when
$\underline{t=0}$ $\quad \underline{t=3/2}$

14. $\boxed{D}$ rate of change of length: $\frac{dL}{dt} = \frac{k}{\sqrt{L}}$ ← inversely proportional
← square root of length

15. $\boxed{C}$ f increasing when $f'(x)>0$ $\quad f' = \frac{3-x}{2xe^x}$

$f'(x)=0$ at $x=3$
$f'(x)$ UD at $x=0$

	-1	$\boxed{0}$	1	$\boxed{3}$	4
f'	$\frac{+}{-} : -$		$\frac{+}{+} : +$		$\frac{-}{+} : -$
f	dec		inc		dec

(16) $\boxed{E}$ slope of the tangent line means $\frac{dy}{dx} = 3x^2-2$

$\int dy = \int 3x^2 - 2\,dx$

$y = \frac{3x^3}{3} - 2x + C = x^3 - 2x + C$ $\qquad$ $5 = 1 - 2 + C$

$5 = -1 + C$

$6 = C$

$y = x^3 - 2x + 6$

(17) $\boxed{D}$ POI means f'' changes signs $\qquad$ make sign chart with f''

* product rule!

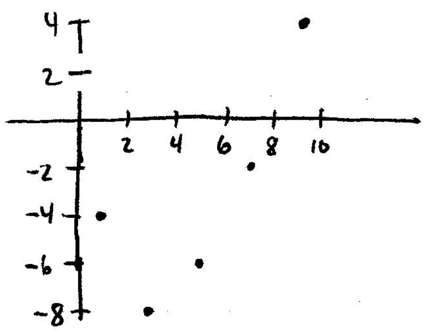
$y = xe^{4x}$ $\qquad$ $y' = x \cdot e^{4x} \cdot 4 + e^{4x} \cdot 1 = e^{4x}(4x+1)$

*product rule again: $y'' = e^{4x}(4) + (4x+1)e^{4x} \cdot 4 = 4e^{4x}[1 + 4x + 1] = 4e^{4x}[4x+2]$

$y'' = 0$ at $x = -2$.

	-3	$\boxed{-2}$ 1
f''	$+(-): -$	$++ : +$
f	conc $\downarrow$	conc $\uparrow$

(18) $\boxed{B}$



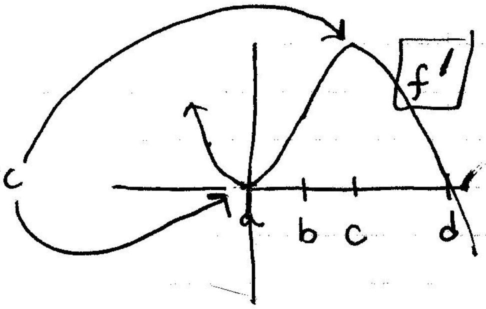
values of $f(x)$ plotted

$f'(x)$ (slope) must be zero on $[1,5]$

since $f(x)$ decreases then increases

(19) $\boxed{E}$ f has point of inflection when

- f changes concavity
- $\boxed{f'}$ changes inc/dec or dec/inc
- f'' changes signs



(20) $\boxed{A}$ find $\frac{dy}{dx}$ at $(2,1)$ implicit differentiation!

product rule! $x\frac{dy}{dx} + y(1) = 4 + 6y\frac{dy}{dx}$ $\qquad$ (sub in $(2,1)$)

$2\frac{dy}{dx} + 1 = 4 + 6\frac{dy}{dx}$

$-3 = 4\frac{dy}{dx}$

$\frac{-3}{4} = dy/dx$

(21) $\boxed{B}$ $g(x) = f^{-1}(x)$ so $f(g(x)) = x$

want to find (take derivative)

$\frac{dg(x)}{dx} = \frac{df^{-1}(x)}{dx}$ at $x=3$

$f'(g(x)) \cdot g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

$g(3) = a$ means

$f(a) = 3$ so $a = 1$

$f(1) = 3$ so $g(3) = 1$

$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \frac{1}{3/2} = 2/3$

(22) $\boxed{C}$ $\frac{dV}{dt} = 12 \frac{in^3}{min}$ WTK: $\frac{dA}{dt}$ when $s = 4$ in A = surface area.

eqns: $V = s^3$ $A = 6s^2$

Deriv: $\frac{dV}{dt} = 3s^2 \cdot \frac{ds}{dt}$ $\frac{dA}{dt} = 12s \cdot \frac{ds}{dt}$

$12 = 3(4^2) \cdot \frac{ds}{dt}$ $\frac{dA}{dt} = 12(4)\left(\frac{1}{4}\right) = 12$

$\frac{1}{4} = \frac{3 \cdot 4}{3 \cdot 4 \cdot 4} = \frac{12}{3 \cdot 16} = \frac{ds}{dt}$

(23) $\boxed{D}$ f is differentiable (given) so f must be continuous which means limits must exist at a.

I. True! This is saying limit exists at $x=a$.

II. False! This is saying the limit (y-value) equals the slope at $x=a$

III. True! $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ is the definition of derivative at $x=a$.

(24) $\boxed{D}$ $y_1 = kx$ and $y_2 = 2^x$ are tangent when they are equal and have equal slope.

$y_1 = y_2$ $y_1' = y_2'$

$kx = 2^x$ $k = 2^x \cdot \ln 2$

$k = \frac{2^x}{x} \xrightarrow{\text{sub in}} \frac{2^x}{x} = 2^x \cdot \ln 2$

$2^x = 2^x \cdot \ln 2 \cdot x$

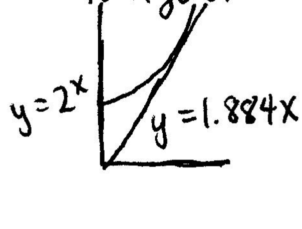
$k = \frac{2^{1.443}}{1.443}$ $\frac{1}{\ln 2} = \frac{2^x}{2^x \cdot \ln 2} = x$

$k = 1.884$ $x = \frac{1}{\ln 2} = 1.443$

Easier way!

graph $y_1 = kx$ $y_2 = 2^x$

sub in k values & see which one makes 2 graphs tangent



(25) $\boxed{B}$ instantaneous rate of change : $f'(x) = \dfrac{f(b)-f(a)}{b-a}$ Average rate of change

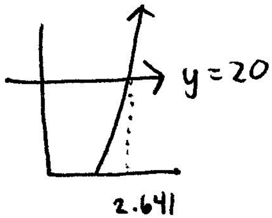
$f(x) = x^3 - 3x^{1/2}$

$f'(x) = 3x^2 - \frac{3}{2}x^{-1/2}$

on $[1,4]$ $\dfrac{f(4)-f(1)}{4-1} = \dfrac{58 - {}^-2}{3} = \dfrac{60}{3} = 20$

graph $y_1 = 3x^2 - \frac{3}{2}x^{-1/2}$ $y_2 = 20$

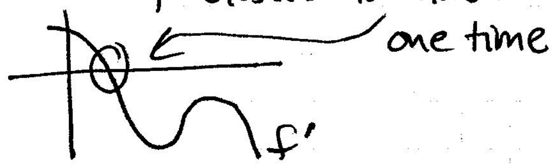
find intersection pt. (2nd/Trace) at $x = 2.641$



(26) $\boxed{B}$ f has relative extrema when f' changes signs, f' crosses x-axis.

Graph $f'(x)$ window $0 < x < 9$

(make sure radian mode)



(27) $\boxed{C}$ $g(x) = \dfrac{x}{f(x)}$ $g'(x) = \dfrac{f(x)\cdot 1 - x f'(x)}{[f(x)]^2}$ (Quotient Rule)

$$g'(4) = \frac{f(4) - 4f'(4)}{[f(4)]^2} = \frac{6-4(3)}{6^2} = \frac{6-12}{36} = \frac{-6}{36} = \boxed{\frac{-1}{6}}$$

(28) $\boxed{E}$ f has a point of inflection when f'' changes signs (graph of f'' crosses x-axis) this only happens at d.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

Chain rule

a) Slope: $K'(x) = f'(g(x))\cdot g'(x)$

$K'(3) = f'(g(3))\cdot g'(3) = f'(6)\cdot 2 = 5\cdot 2 = 10$

Point: $K(3) = f(g(3)) = f(6) = 4$ Line: $y - 4 = 10(x-3)$

Stop here

b) $h'(x) = \dfrac{f(x)g'(x) - g(x)\cdot f'(x)}{f(x)^2}$ $h'(1) = \dfrac{f(1)g'(1) - g(1)\cdot f'(1)}{f(1)^2} = \boxed{\dfrac{-6(8) - 2(3)}{(-6)^2}} = \dfrac{-54}{36}$

stop here

c) $\int_1^3 f''(2x)\,dx = \int_2^6 f''(u)\,\frac{1}{2}\,du = \frac{1}{2}f'(u)\Big|_2^6 = \frac{1}{2}f'(6) - \frac{1}{2}f'(2) = \boxed{\frac{1}{2}(5) - \frac{1}{2}(-2)}$

$= \frac{5}{2} + \frac{2}{2} = \frac{7}{2}$

$u = 2x$ $x=3 \rightarrow u=6$

$x=1 \rightarrow u=2$

$\frac{du}{dx} = 2$

$\frac{1}{2}du = dx$

OR fast u-sub: $\int_1^3 f''(2x)\,dx = \frac{1}{2}f'(2x)\Big|_1^3 = \frac{7}{2}$

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is $\boxed{\text{vertical}}$.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

a) point: $(-1,1)$
Slope: $\frac{dy}{dx} = \frac{1}{3-(-1)} = \frac{1}{4}$ $\qquad y-1=\frac{1}{4}(x+1)$

b) $\frac{dy}{dx}$ DNE so $3y^2-x=0$
$3y^2=x$

$y^3-xy=2$
$y^3-(3y^2)y=2$
$y^3-3y^3=2$
$-2y^3=2$
$y^3=-1$
$y=-1$

$(-1)^3-x(-1)=2$
$-1+x=2$
$x=3$
$\boxed{(3,-1)}$

c) $\frac{dy}{dx}=\frac{y}{3y^2-x}$ $\qquad \frac{d^2y}{dx^2}=\frac{(3y^2-x)\frac{dy}{dx}-y\left(6y\frac{dy}{dx}-1\right)}{(3y^2-x)^2}$

Stop here

at $(-1,1)$ $\frac{dy}{dx}=\frac{1}{4}$ so $\frac{d^2y}{dx^2}=\boxed{\frac{(3-(-1))\frac{1}{4}-1\left(6\cdot\frac{1}{4}-1\right)}{(3-(-1))^2}}=\frac{1}{32}$